DEPARTMENT OF REVENUE:
ASSESSMENT/SALES RATIO STUDIES

PROGRAM EVALUATION DIVISION
OFFICE OF THE LEGISLATIVE AUDITOR
STATE OF MINNESOTA

May 31, 1978
The Program Evaluation Division of the Legislative Audit Commission was established by Chapter 204, Section 91 of the Laws of Minnesota for 1975. The Division is authorized to "determine the degree to which activities and programs entered into or funded by the state are accomplishing their goals and objectives, including an evaluation of goals and objectives, measurement of program results and effectiveness, alternative means of achieving the same results, and efficiency in the allocation of resources." This evaluation, Department of Revenue: Assessment/Sales Ratio Studies, is the seventh undertaken by this Division.

The Legislative Audit Commission directed the Program Evaluation Division to review the procedures used by the Department of Revenue in their assessment/sales ratio studies, and particularly to evaluate the results of their studies with respect to accuracy and statistical validity. The primary issues addressed in this evaluation are:

1. whether the Department's procedures produce acceptable indicators of average assessment levels and dispersion in assessment/sales ratios; and

2. whether the average assessment/sales ratios are acceptable for use as equalization factors in state aid formulas.

For each report, a uniform review procedure is followed. After a preliminary draft is completed, it is submitted to the agency evaluated for verbal and written comments. The written responses of the Minnesota Department of Revenue are included in the appendices. In addition, the report is reviewed by a subcommittee of the Legislative Audit Commission prior to its release. For this report the subcommittee consisted of Representative William N. Kelly, chairman of the House Tax Committee, and Senator William McCutcheon, chairman of the Senate Tax Committee. We are most grateful for their helpful advice and direction.

We thank Arthur C. Roemer, the Commissioner of Revenue, and his staff for their valuable time and assistance on this project.

Edward Burek was project manager and author of the report. Leif Hartmark acted as project consultant and made suggestions on the conduct of the research. Scheffel Wright reviewed various drafts.

May 31, 1978

Bruce Spitz
Deputy Legislative Auditor
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EXPLANATION OF TERMS

(1) **Assessor's market values:** There are two types of assessor's market value—estimated market value and limited market value.

(a) **Estimated market value:** This is the assessor's estimate of what the property would sell for in the market. Sales ratios calculated by dividing the estimated value by the sale price are used as a measure of the assessor's performance or accuracy. They are not used directly for school aid purposes.

(b) **Limited market value:** The limited market value is a product of a statutory limitation enacted in 1973. In essence it limits any annual increase in an individual assessment to 10 percent of the previous year's limited value or 25 percent of the difference between the previous limited value and the new estimated value. Since limited values are used for tax purposes, the sales ratios used for school aid purposes are based on limited values.

(2) **Classification ratio:** The classification ratio is the fraction of limited market value against which taxes are levied. The legislature has generally established a different classification ratio for each property type for the purpose of distributing the property tax burden among property types according to legislative intent.

(3) **Assessed value:** Any property tax levied against a unit of property is the product of the mill levy (tax rate) times the assessed value. The assessed values are derived by multiplying the limited market values by the classification ratio for that property type. For example, the assessed value of a unit of non-homestead residential property is 40 percent of its limited market value. The mill rate is then applied against this assessed value.

(4) **Adjusted assessed value:** This is the "assessed" value described in (3) above which has been equalized by dividing by aggregate assessment/sales ratios. Since local taxes are based on assessed values, the adjusted assessed value (equalized assessed value) is the truest measure of local property wealth for tax and school aid purposes. It shows how much the assessed value would be if property were valued at 100 percent of its value in the marketplace.

(5) **Indicated market value:** The indicated value is the limited market value divided by the aggregate assessment/sales ratio. In effect, this is an equalized value since it shows what the limited value would be if the property were valued at 100 percent of its value in the marketplace.

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1 The aggregate assessment/sales ratio is a form of average assessment/sales ratio used in the determination of school foundation aids. See page 4.
SUMMARY OF FINDINGS AND RECOMMENDATIONS

This evaluation focuses on two basic issues:

(1) the accuracy of the calculated average assessment/sales ratios and dispersion indicators produced by Department of Revenue procedures; and

(2) the acceptability of calculated average assessment/sales ratios for use as equalization factors in state aid formulas.

Findings and recommendations relating to these issues are summarized below. Supporting documentation for all findings and recommendations can be found in the report or are available from the Program Evaluation Division.

EFFECTS OF THE PRESENT METHOD OF DATA MATCHING

Individual assessment/sales ratios are calculated by dividing the assessor's market value for a given parcel by its sale price. The data used for Department of Revenue assessment/sales ratio studies generally consists of three years of sales, matched with the most recent assessor's market values of the properties. For example, in the 1977 study, sales occurring in 1975, 1976, and 1977 are all matched with 1977 assessor's market values.

FINDINGS:

(1) The average assessment/sales ratios and dispersion measures calculated by the Department of Revenue overstate the true levels. Serious biases are caused by the present system of data matching. Specifically:

(a) If there is inflation, the average assessment/sales ratios and their dispersion tend to be overstated.

(b) Different rates of inflation will result in different biases in the averages and dispersion measures.

(2) Due to the present system of data matching, the calculated averages are sensitive to the number and volume of sales in each year of the study. An unusually high volume of sales in a given year will cause a "ripple effect" in the averages over
time. As these sales enter the sample the averages will fall; then they will increase until these sales leave the three-year sample, at which time the averages will again fall.

(3) As indicated in (1) and (2) above, the calculated averages will overstate the true values by disproportionate amounts and will be subject to unwarranted variation over time. This will distort the allocation of school aids and produce indicators which are not acceptable for use in reassessment efforts.

RECOMMENDATION:

(1) All sales (or appraisals) should be matched with assessor's market values for the year of sale (or appraisal). This is the fundamental step which must be taken if any significant improvement is to occur in the sales ratio studies.

SCREENING PROCEDURES

The basic data are screened at the Department of Revenue to eliminate transactions which are not between impartial buyers and sellers. Under the present system of data matching, where all three years of sales are matched with the newest assessor's market values, it is necessary to update all data in the sample each year. Two computerized edits follow this updating. The "possible errors run" checks for invalid and incorrect data. The "extreme ratio listing" prints all ratios lying 25 points or more from the median; these can be re-examined.

FINDINGS:

(1) The present system of data matching, where all sales are matched with the newest assessor's market values, impairs the quality of the screening process.

(a) Since the data updating must be performed, all data must be screened annually. The data updating phase diverts time and resources which could be devoted to data screening, and the amount of data requiring screening is greatly increased.

(b) The present data matching system impairs the effectiveness of the extreme ratio listing. It causes old sales to predominate in the high extremes merely because they are old and new sales to predominate in the low range merely because they are new.
(2) The quality of the screening is not consistent, varying among counties.

RECOMMENDATIONS:

(1) As recommended previously, all observations should be matched with assessor's market values in the year of sale. This would permit the following changes which improve the quality of the screening procedures:

(a) Only the newest year's data would need to be screened. The amount of data to be scrutinized annually would decrease to approximately one-third its current level.

(b) The present practice of updating older sales could be eliminated. Eliminating this phase will permit data editing to begin earlier, and to be performed more intensively.

THE ADJUSTED ASSESSED VALUE CALCULATION

The adjusted assessed value is defined as the assessed value that would be obtained if the assessor were valuing all property at its full value in the market. The essence of the procedure for calculating these values is to divide the total assessed value by an aggregate assessment/sales ratio. The adjusted assessed value is used to indicate a community's taxable property wealth and is a key factor in determining the amount of school aids received by the community. The school aid formula is designed so that the higher the adjusted assessed value, the more funds must be raised from local taxes. Communities with comparable adjusted assessed values raise equal amounts of local tax revenues for school funding.

FINDINGS:

(1) The procedure now used for calculating adjusted assessed values yields inaccurate results. The Department's method will yield an accurate total only if all classification ratios are the same, or if the average assessment/sales ratios are identical for all property types. Neither of these conditions is true in practice.

(2) The present procedure for calculating adjusted assessed values is inconsistent with legislative intent to spread the property tax burden through use of classification ratios.
RECOMMENDATION:

(1) Given adoption of the data matching recommendation, to calculate adjusted assessed values a sales-value-weighted average of total assessed value should be divided by the appropriate aggregate assessment/sales ratio for each property type and municipality. The number of years of total assessed values used in the calculation should match the number of years used in calculating the aggregate assessment/sales ratio.

THE GENERAL NEED FOR STATISTICAL ANALYSIS

Several sections of this report employ statistical procedures to test the admissibility of certain data for the studies. The recommendations associated with these sections advise using statistical tests when the validity of data is questionable. To implement these recommendations and to develop additional procedures where necessary, the Department will require staff competent in advanced statistical techniques. In light of these substantial needs, we make the following general recommendation:

RECOMMENDATION:

(1) The Department of Revenue should hire additional personnel to implement statistical testing procedures relating to the design and uses of assessment/sales ratio studies.

THE USE OF APPRAISALS

In some cases where the sample size has been deemed insufficient, the Department of Revenue has enlarged the sample by using appraisals. These appraisals are then used to form individual assessment/appraisal ratios which are added to the sample of assessment/sales ratios.

If these assessment/appraisal ratios are comparable to assessment/sales ratios, then enlarging the sample through use of appraisals will improve the estimate of the average sales ratios and improve the quality of the studies. If the individual assessment/appraisal ratios are not comparable to individual assessment/sales ratios, then the resulting averages and adjusted assessed values will be distorted.
FINDINGS:

(1) Statistical tests performed by the Program Evaluation Division indicate that in some cases the use of appraisals in past studies was detrimental, resulting in poorer estimates of average assessment/sales ratios. In 18 counties the sample size was insufficient to generate a test result. Of the 69 testable counties, nearly one-third (21 counties) had a less than 20 percent probability of being comparable. In 11 of these 21 counties, the probability of being comparable was less than 10 percent.

The following recommendations are contingent upon the adoption of our earlier recommendation concerning data matching. The recommendations below should not be followed if the present system of data matching is continued.

RECOMMENDATIONS:

(1) The Department of Revenue should develop and implement a test procedure to determine the advisability of using a particular set of appraisals. An alternative, the use of a fourth year of sales, should also be tested. A set of guidelines for testing appears in Chapter IV of the report.

(2) The Department of Revenue should develop a rigorous procedure to determine where sample size should be increased, and by how much the sample should be expanded.

COMBINING COMMERCIAL AND INDUSTRIAL PROPERTY

Prior to the 1976 sales ratio study, data for commercial and industrial property were maintained separately, and average assessment/sales ratios were calculated for each category. Beginning with the 1976 study, however, the two property types were combined and only one set of average assessment/sales ratios was calculated for the combined category.

If the two property types are comparably assessed, no impairment results from combining them into a single category. However, if the property types are not comparably assessed, the following problems may occur:

(1) The adjusted assessed values and aid distribution will be distorted.
(2) The calculated adjusted assessed values and aids may fluctuate over time, even if actual property wealth is constant.

(3) The use of appraisals, even appraisals which are comparable to sales, can alter the aid allocation.

FINDINGS:

(1) Tests performed by the Program Evaluation Division on 66 counties (21 counties could not be analyzed due to insufficient sample size) revealed that 23 counties had less than 20 percent probabilities of comparability between commercial and industrial assessment/sales ratios. These tests included appraisals in both the industrial and commercial categories.

(2) When commercial appraisals were eliminated and another series of tests was performed, only 46 counties could be analyzed. Of these, 9 counties had less than a 20 percent probability of comparability between commercial and industrial assessment/sales ratios.

RECOMMENDATIONS:

(1) Since even conservative statistical tests suggest that commercial and industrial properties cannot validly be merged for many counties, these categories should generally be maintained and processed separately for assessment/sales ratio and aid calculation purposes.

AGRICULTURAL PROPERTY

There are currently two classifications used for agricultural property. The first classification, improved agricultural land, is farmland with buildings present. The second category, unimproved agricultural land, refers to tracts of farmland on which no buildings are present.

The current practice is to combine these two categories in the study, which is also the procedure for commercial and industrial property. If the two agricultural categories do not have comparable assessment/sales ratios, the following problems may occur:

(1) The calculated average assessment/sales ratios for the combined category may fluctuate, causing the calculated adjusted assessed values and aids to vary.
(2) Counties with comparable property wealth may not have similar calculated adjusted assessed values, leading to a distorted aid allocation.

FINDINGS:

(1) Based upon statistical tests, in nearly half the counties in the state there is evidence that improved agricultural sales and unimproved sales have different average assessment/sales ratios.

(2) The differences between the assessment/sales ratios for the two categories appear to be due to differences in assessment levels between land and buildings.

The problems presently encountered with agricultural properties in the assessment/sales ratio studies have no simple solutions. Until the feasibility and accuracy of alternative procedures suggested in this report can be determined, there is no better alternative than to continue present procedures.
INTRODUCTION

Assessment/sales ratio studies are conducted annually by the Department of Revenue and provide detailed information on the level of property assessment in each municipality. The basic data used for the sales ratio study are individual assessment/sales ratios, obtained by dividing the assessor's market value for a given parcel by the sale price of that property. Average assessment levels and the dispersion of assessment/sales ratios are calculated from this information. These data come from the Certificate of Real Estate Value, which is filed with the County Auditor when real estate is sold in Minnesota.

Given the uses of these studies, their quality and accuracy is of crucial importance for these reasons:

(1) The information provided by assessment/sales ratio studies is used to ensure that property tax burdens are equitable and consistent with legislative intent. The Commissioner of Revenue, acting as the State Board of Equalization, uses the information to reduce large discrepancies in assessment levels between and within counties.

(2) The information can also be used by local assessors to indicate how consistent each assessor is, to make comparisons between assessors, and to detect geographic areas and property types which require more concerted assessment efforts.

(3) The average assessment/sales ratios calculated for each property type within each community are used as "equalizing factors" or correction factors in various state aid formulas. The school foundation aid formula is the most significant formula which uses these ratios. Currently over $1.2 billion in school foundation aids is distributed each biennium.

The Legislative Audit Commission directed the Program Evaluation Division to review the procedures used by the Department of Revenue in its assessment/sales ratio studies, and particularly to evaluate the results of these studies with respect to accuracy and statistical validity. The primary issues addressed by this evaluation are:

(1) whether the Department's procedures produce acceptable indicators of average assessment levels and dispersion in assessment/sales ratios; and
whether the average assessment/sales ratios are acceptable for use as equalization factors in state aid formulas.

The staff members of the Department of Revenue were extremely helpful and receptive to our study. Many of the problem areas covered in this report were brought to the attention of the Program Evaluation Division by Department of Revenue personnel. Staff members spent many hours describing the complex procedures used in the assessment/sales ratio study and explaining strengths and weaknesses of various stages.

In the course of our review, it became apparent that tailoring the sales ratio studies to one form of use would diminish potential effectiveness for other uses. In this report, our recommendations attempt to enhance characteristics of the average ratios which produce desirable results when these ratios are used in school aid formulas. Due to this emphasis, some recommendations may appear odd to both the layman and the statistically sophisticated. Documentation justifying many of the recommended procedures is available from the Program Evaluation Division.

The first chapter in this report analyzes the present method of matching assessor's market values with sale prices to produce individual assessment/sales ratios. Chapter II deals with screening procedures designed to eliminate improper data from the sample. Chapter III discusses flaws in the procedure for calculating adjusted assessed values, an element of the school aid formula which is used to estimate local property wealth. Chapter IV contains several sections relating to the need for personnel trained in statistical techniques, the use of appraisals, the implications of combining commercial and industrial property in the studies, problems in the agricultural sample, and possible flaws in computer programs and decision rules for selecting average assessment/sales ratios to be used in the preliminary stages of school aid computations. The concluding chapter suggests broad guidelines for tailoring a study to provide indicators of assessor performance, discusses the need for parallel studies, and suggests areas for future research.

TYPES OF ASSESSMENT/SALES RATIO CALCULATIONS

The basic data are individual assessment/sales ratios—the assessor's market value for each parcel divided by the sale price of

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1 Care has been taken to ensure that recommendations are basically compatible with the various uses of the Department of Revenue's studies. There is a definite benefit, however, in performing several studies, each tailored to a particular purpose.
the parcel. The averages calculated from this information are the arithmetic mean, the median, and the aggregate mean. These measures are calculated for each property type in each municipality, where sample size permits. The arithmetic mean is obtained by summing the individual ratios and dividing by the number of individual ratios in the sample. The arithmetic mean is a good indicator of the "typical" assessment/sales ratio for the given community and property type. The median is obtained by ranking the ratios from lowest to highest and selecting the middle ratio. The aggregate mean is a more sophisticated measure, obtained by summing the assessor's market values in each sample, summing the sale prices in each sample, and then dividing the sum of the assessor's market values by the sum of the sale prices. The aggregate mean is used as an equalizing factor in aid formulas and is generally the best form of average for this purpose.

The calculated measures of dispersion are the price related differential (index of regression), the coefficient of dispersion, the standard deviation, and the coefficient of variation. The price related differential is used to indicate whether there is a difference in the relative assessment level between high-priced and low-priced properties. The other dispersion measures are more general in purpose and are designed to measure dispersion in individual assessment/sales ratios both within and between value ranges. The lower the value of these measures, the more uniform the individual assessment/sales ratios and hence, the more consistent the individual assessor's performance.

The following numerical examples illustrate the statistics described above.

1. INDIVIDUAL ASSESSMENT/SALES RATIO

For each sale in the sample, the individual assessment/sales ratio is simply the assessor's market value divided by the sale price.

Example:

<table>
<thead>
<tr>
<th>Parcel</th>
<th>Assessor's Market Value</th>
<th>Sale Price</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$20,900</td>
<td>$19,000</td>
<td>110.0%</td>
</tr>
<tr>
<td>2</td>
<td>28,500</td>
<td>30,000</td>
<td>95.0%</td>
</tr>
<tr>
<td>3</td>
<td>22,950</td>
<td>25,000</td>
<td>90.0%</td>
</tr>
<tr>
<td>4</td>
<td>33,200</td>
<td>41,500</td>
<td>80.0%</td>
</tr>
<tr>
<td>5</td>
<td>31,200</td>
<td>52,000</td>
<td>60.0%</td>
</tr>
</tbody>
</table>

1These numerical examples are taken directly from the introduction to the Department of Revenue's Real Estate Assessment/Sales Ratio Study.
2. MEAN ASSESSMENT/SALES RATIO

The mean, also called the arithmetic average, is a measure of central location. The mean is found by adding the individual assessment/sales ratios and then dividing by the number of individual ratios in the sample.

Example:

<table>
<thead>
<tr>
<th>Parcel</th>
<th>Assessor's Market Value</th>
<th>Sale Price</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$20,900</td>
<td>$19,000</td>
<td>110.0%</td>
</tr>
<tr>
<td>2</td>
<td>28,500</td>
<td>30,000</td>
<td>95.0%</td>
</tr>
<tr>
<td>3</td>
<td>22,950</td>
<td>25,000</td>
<td>90.0%</td>
</tr>
<tr>
<td>4</td>
<td>33,200</td>
<td>41,500</td>
<td>80.0%</td>
</tr>
<tr>
<td>5</td>
<td>31,200</td>
<td>52,000</td>
<td>60.0%</td>
</tr>
</tbody>
</table>

\[ \text{Mean} = \frac{87.0\%}{5} = 435.0\% \]

3. MEDIAN

The median is also a measure of central tendency. It is found by arranging the individual assessment/sales ratios from smallest to largest, then selecting the middle ratio in the series.

Example:

<table>
<thead>
<tr>
<th>Parcel</th>
<th>Assessor's Market Value</th>
<th>Sale Price</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$20,900</td>
<td>$19,000</td>
<td>110.0%</td>
</tr>
<tr>
<td>2</td>
<td>28,500</td>
<td>30,000</td>
<td>95.0%</td>
</tr>
<tr>
<td>3</td>
<td>22,950</td>
<td>25,000</td>
<td>90.0%</td>
</tr>
<tr>
<td>4</td>
<td>33,200</td>
<td>41,500</td>
<td>80.0%</td>
</tr>
<tr>
<td>5</td>
<td>31,200</td>
<td>52,000</td>
<td>60.0%</td>
</tr>
</tbody>
</table>

\[ \text{Median} = 90.0\% \]

4. AGGREGATE ASSESSMENT/SALES RATIO

The aggregate mean is computed by dividing the sum of assessor's market values for the properties sold by the total sale prices of those properties. In the aggregate mean, each property sold is effectively given a weight proportionate to its sale price. Higher priced properties have more weight than lower priced properties in the determination of this average.
Example:

<table>
<thead>
<tr>
<th>Parcel</th>
<th>Assessor's Market Value</th>
<th>Sale Price</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$20,900</td>
<td>$19,000</td>
<td>110.0%</td>
</tr>
<tr>
<td>2</td>
<td>28,500</td>
<td>30,000</td>
<td>95.0%</td>
</tr>
<tr>
<td>3</td>
<td>22,950</td>
<td>25,500</td>
<td>90.0%</td>
</tr>
<tr>
<td>4</td>
<td>33,200</td>
<td>41,500</td>
<td>80.0%</td>
</tr>
<tr>
<td>5</td>
<td>31,200</td>
<td>52,000</td>
<td>60.0%</td>
</tr>
<tr>
<td></td>
<td>$136,750</td>
<td>$168,000</td>
<td></td>
</tr>
</tbody>
</table>

81.4% aggregate sales ratio

$168,000 \sqrt{136,750.00}$

MEASURES OF DISPERSION

The following measures are used as indicators of dispersion:

5. PRICE RELATED DIFFERENTIAL (INDEX OF REGRESSION)

The index of regression is an indicator of "vertical dispersion" (i.e., it indicates whether there is a difference in assessment levels between low-priced vs. high-priced properties). Thus the index is used to determine if the assessor's performance is consistent across property value ranges. The index is formed by dividing the mean assessment/sales ratio by the aggregate sales ratio, then multiplying by 100. The mean assessment/sales ratio is a simple arithmetic average of all individual ratios. The aggregate mean, however, has the property of weighting higher priced sales more heavily than lower priced sales. Therefore, if high-priced sales tend to have lower sales ratios than lower priced sales, the aggregate mean will be less than the arithmetic mean. If the index is greater than 100 (in other words, if the arithmetic mean is greater than the aggregate ratio), then high-priced property is under-assessed relative to lower priced property. Using the aggregate ratio from the earlier example of 81.4 percent, and the arithmetic mean of 87 percent, the index is 106.9.

6. THE COEFFICIENT OF DISPERSION

The coefficient of dispersion is a measure of the variability or dispersion of individual assessment/sales ratios in the sample. To calculate the coefficient of dispersion:
a. Find the difference between each individual assessment/sales ratio and the median ratio, then add all the differences ignoring plus or minus signs.

b. Divide this sum by the number of properties in the sample.

c. Divide the result in (b) by the median ratio and multiply this result by 100.

This result is the coefficient of dispersion, or "index of assessment inequality." The lower the coefficient of dispersion, the more uniform are the assessments.

Example:

<table>
<thead>
<tr>
<th>Ratio</th>
<th>Deviation From Median</th>
</tr>
</thead>
<tbody>
<tr>
<td>110.0%</td>
<td>20.0%</td>
</tr>
<tr>
<td>95.0%</td>
<td>5.0%</td>
</tr>
<tr>
<td>90.0% median</td>
<td>0.0%</td>
</tr>
<tr>
<td>80.0%</td>
<td>10.0%</td>
</tr>
<tr>
<td>60.0%</td>
<td>30.0%</td>
</tr>
<tr>
<td></td>
<td>65.0% total deviation</td>
</tr>
</tbody>
</table>

b. \[ \frac{13.0\%}{5} \text{ average deviation} \]
\[ \frac{65.0\%}{65.0\%} \text{ total deviation} \]

c. \[ \frac{.144}{90\%} \text{ 13.0\% average deviation} \]
\[ .144 \times 100 \]
\[ 14.4 \text{ coefficient of dispersion} \]

7. THE STANDARD DEVIATION

The standard deviation is a measure of dispersion measuring the variability of individual assessment/sales ratios in relation to the mean ratio. The computational steps are:

a. Find the arithmetic mean ratio for the sample.

b. Find the difference between each individual assessment/sales ratio and the arithmetic mean.

c. Square these deviations (multiply each deviation times itself), then sum the results.

d. Divide the sum in (c) by the number of properties in the sample.

e. Compute the square root of the result in (d).
Example:

<table>
<thead>
<tr>
<th>Ratio</th>
<th>Deviation From Mean</th>
<th>Deviation Squared</th>
</tr>
</thead>
<tbody>
<tr>
<td>110.0%</td>
<td>23.0</td>
<td>529.0</td>
</tr>
<tr>
<td>95.0%</td>
<td>8.0</td>
<td>64.0</td>
</tr>
<tr>
<td>90.0%</td>
<td>3.0</td>
<td>9.0</td>
</tr>
<tr>
<td>80.0%</td>
<td>-7.0</td>
<td>49.0</td>
</tr>
<tr>
<td>60.0%</td>
<td>-27.0</td>
<td>729.0</td>
</tr>
<tr>
<td>435.0%</td>
<td></td>
<td>1,380.0</td>
</tr>
</tbody>
</table>

\[
\text{mean} = \frac{87.0}{\sqrt{\frac{276.0}{5}}} = 16.6
\]

8. THE COEFFICIENT OF VARIATION

The coefficient of variation standardizes the standard deviation so that comparisons of relative variability can be made. To calculate the coefficient of variation, merely divide the standard deviation by the arithmetic mean, then multiply by 100. In our example, dividing the standard deviation (16.6) by the mean (87.0) and multiplying by 100 yields a coefficient of variation of 19.1.

OVERVIEW OF THE DEPARTMENT OF REVENUE'S PROCESS

At the Department of Revenue, the Certificates of Real Estate Value receive a manual screening. This is the first of several screening procedures designed to eliminate transactions which are not "arm's-length." The term "arm's-length" means transactions between willing, impartial buyers and sellers. If the sale price given on the certificate is to be an indicator of the property's value on the open market, sales between related individuals, sales forced due to impending foreclosure, and other questionable sales must be eliminated from the study.

The incoming data which survive the manual screening are then sorted into categories by property type and geographic area. The major categories are residential, commercial and industrial, apartment, seasonal recreational, improved agricultural land, and unimproved agricultural land.
Due to the low number of sales for several property types in outstate regions, the Department's sales ratio studies use a multi-year sample. In the studies used for aid distribution, the procedure is to match three years of sales with the most recent assessor's market values for these properties. For example, in the 1977 study there were sales from 1975, 1976, and 1977. All these sales were matched with 1977 assessor's market values.

The current system of data matching therefore requires that the newest assessor's market values be obtained for all older sales in the sample. Thus all data in the multi-year sample must be updated annually. Following this updating, the data are keypunched and subjected to several computer edits, designed to supplement the manual screening process and to catch any errors introduced in the updating and keypunching phases.

The acceptable data are then used to calculate the averages and measures of dispersion illustrated on pages 2 through 7 for the various property types in each municipality in the state, where sample size permits. The Department of Revenue publishes this information in its Real Estate Assessment/Sales Ratio Study. The information is also stored for further use in the computerized stages of school aid determination.

Foundation school aids are based on a measure of local taxable property wealth. In determining the amount of funding which must be raised locally, it is first necessary to accurately estimate local taxable property wealth. The total assessed values are unacceptable, because they may differ from one district to another due solely to different assessment performance. For example, if one district's assessor is valuing property at 50% of its value in the marketplace, while in another district the assessor is valuing property much closer to actual market value, then the total assessed values for the districts may be very different, in spite of comparable property wealth. To correct this problem the assessed values are adjusted by dividing the total assessed values by average assessment/sales ratios, producing adjusted assessed values. This procedure should determine what the assessed value would be if the assessor were assessing property at 100 percent of its value in the marketplace. Once the community's taxable property wealth (the adjusted assessed value) has been identified, a uniform mill rate is applied against this value to determine the level of local revenue required. The difference between this local effort and the total

---

1 This is the typical procedure, although there are deviations from this norm. Occasionally four years of data have been used for certain property types. Also, the sales data are often supplemented with appraisals in situations where the Department feels there are insufficient sales to calculate meaningful average ratios.

2 This system of data matching is analyzed in Chapter I.
amount required to meet the per pupil guarantee is the amount of state aids to which the district is entitled. The aid formula is structured so that the higher the adjusted assessed value (the wealthier the community), the greater the amount of local tax revenues which must be raised, and the lower the school aids to that community. Comparable communities would be required to raise equal amounts of local tax revenues for school funding.

1The support per pupil unit is established by the Legislature. In 1976-1977, $960 per pupil unit was required. For 1977-1978, $1,030, and for 1978-1979, $1,090 per pupil unit is required.
CHAPTER I

THE PRESENT METHOD OF DATA MATCHING

The data used for the Department of Revenue's annual assessment/sales ratio studies generally consists of three years of sales matched with estimated market values and limited market values for the properties. For example, in the 1977 study, sales from 1975, 1976, and 1977 were all matched with 1977 estimated market values. This system of data matching will produce distorted estimates of the dispersion of assessment/sales ratios (dispersion) and average assessment levels whenever property values are inflating. The following simplified example illustrates the effect on both average levels and dispersion.

In the example illustrated in Exhibit 1, it is assumed that a particular unit of property sold in 1975 for $20,000, and that in 1975 the assessor claimed this property was worth $16,000, or 80 percent of its sale price ($16,000 ÷ $20,000 = 80%). The 1975 estimated market value is the assessor's official estimate of this property's value in 1975. In the 1977 assessment/sales ratio study, however, this sale would be matched with the 1977 estimated market value, which is the assessor's estimate of this parcel's value in 1977. If there is inflation, the assessor will be increasing the estimated values. If property prices are inflating at 15 percent per year, and the assessor is keeping pace with inflation, the 1977 estimated market value on this parcel will be $21,160 ($16,000 × 1.15 × 1.15 = $21,160). This 1977 assessment will be matched with the 1975 sale price of $20,000, and in the 1977 study the parcel will appear to be assessed at 105.8 percent ($21,160 ÷ $20,000 = 105.8%).

Actually, two studies are performed. One assessment/sales ratio study uses ratios based upon estimated market values, the other is based upon limited market values. As mentioned in the Explanation of Terms, the estimated market value is simply the assessor's estimate of what the property would sell for in the market. Sales ratios calculated on the estimated value compared to sale price are used as a measure of the assessor's performance or accuracy. The limited market value is the assessor's estimate of market value reduced by legal constraints on the amount of annual increase permitted. The limited value may not increase by more than 10 percent of the previous year's limited value or 25 percent of the difference between the previous limited value and the present estimated value, whichever is greater. Property taxation is based upon limited values. Consequently, the sales ratios used for school aid purposes are based upon these values.

The distorting effect of the present system of data matching upon average assessment/sales ratios and dispersion measures has been empirically verified by recalculating the results found in the Department's Real Estate Assessment/Sales Ratio Study. The data used by the Program Evaluation Division matched sale prices with assessor's market values in the year of sale, as recommended in this chapter.
EXHIBIT 1

EFFECT OF PRESENT DEPARTMENT OF REVENUE MATCHING PROCEDURES ILLUSTRATED ASSUMING 15% INFLATION

<table>
<thead>
<tr>
<th></th>
<th>1975 SALE</th>
<th></th>
<th>1976 SALE</th>
<th></th>
<th>1977 SALE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PRICE</td>
<td>E.M.V.</td>
<td>E.M.V.</td>
<td>PRICE</td>
<td>E.M.V.</td>
</tr>
<tr>
<td></td>
<td>$20,000</td>
<td>$16,000</td>
<td>$21,160</td>
<td>$30,000</td>
<td>$24,000</td>
</tr>
<tr>
<td></td>
<td>($16,000 × 1.15 × 1.15)</td>
<td>($24,000 × 1.15)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>80%</td>
<td>105.8%</td>
<td></td>
<td>80%</td>
<td>92%</td>
</tr>
</tbody>
</table>

*E.M.V. = ESTIMATED MARKET VALUE*
However, due in part to inflation, comparing a 1975 sale price with a 1977 assessor's market value does not provide a valid indication of the assessment level.

Suppose that in 1976 another property sold for $30,000, which was assessed at $24,000 when sold. The ratio of the estimated market value to sale price, when sold, is again 80 percent ($24,000 ÷ $30,000 = 80%). With 15 percent inflation, this property is valued by the assessor at $27,600 in 1977 ($24,000 x 1.15 = $27,600) and is handled as though it were assessed at 92 percent ($27,600 ÷ $30,000 = 92%) in the 1977 study.

If a 1977 sale occurs, also assessed at 80 percent when sold, this sale would be included in the study with an 80 percent individual ratio.

Since all properties in the sample were assessed at 80% of sale price when matched with the official estimated market value when the sale occurred, this hypothetical assessor is actually assessing at a consistent 80% level, and there is no dispersion. However, when the data are matched using the Department's methodology, it appears as though the sales have three different assessment/sales ratios, 105.8, 92, and 80 percent. The average of these numbers is greater than 80 percent, and thus the average is overstated; also since the individual ratios are not identical, there appears to be dispersion. Furthermore, higher rates of inflation tend to bias the means even more and increase the apparent dispersion. To keep pace with a higher rate of inflation the assessor would increase estimated market values even more, causing older sales to have even higher ratios when matched with the latest estimated market values. The calculated mean would increase and the gap between individual ratios for old vs. new sales would widen, increasing the apparent dispersion.

This argument implies that districts with comparable assessment performance, but with different rates of property inflation, will not have comparable average assessment/sales ratios. As a consequence, a district's adjusted assessed value cannot be accurately determined, and comparable communities may receive disproportionate aids.

1The problems mentioned in the text, and other problems, are illustrated in greater detail in Appendix A.

2Given two districts with identical frequency of reassessment, the district with the higher rate of property inflation will have averages with a higher bias.
Using the information illustrated\(^1\) another serious flaw can be demonstrated. Not only does the current system of data matching result in erroneous average assessment/sales ratios, it also can cause them to fluctuate over time, even when assessor performance is unchanging. This will artificially induce fluctuations in aids. A year with a disproportionately large number of sales (or appraisals) will at first lower the calculated averages; then over time, the averages will increase as these sales (or appraisals) are matched with newer assessments. Finally, when these cases leave the sample, the averages will fall.\(^2\) Referring to Exhibit 1, note that if several more sales had occurred in 1975, all assessed at 80 percent in 1975, their individual assessment/sales ratios would be over 100 percent in the 1977 study, and the average assessment/sales ratio would increase considerably. On the other hand, if more sales had occurred in 1977, they would enter the study with 80 percent ratios, and the averages would fall.

Even in a non-inflationary period, the present data matching procedure produces questionable indicators. Matching old sales with new assessments means that for many sales in the sample the assessor knows the sale price to which his estimate will be compared before he establishes the new estimated (and limited) market value for the property. The average assessment/sales ratios produced from this data are used as an estimate of the level of assessment for all property in the community, both sale properties and non-sale properties. The possibility exists for increasing the apparent average ratios by selectively reassessing properties that have recently been sold ("chasing sales"), rather than increasing the level of assessor's market values for all property in the community. If there is sale chasing, the estimate of the community average assessment/sales ratio (i.e., the averages calculated from the sales ratio sample) will be overstated, and the resulting school aids to the district will be unjustly high. This potential problem of sale chasing exists in inflationary periods as well: inflation biases the average assessment/sales ratios upward, and sale chasing would increase this bias. Thus the present system of data matching

\(^1\) Generally it would not be possible to increase limited market values by 15 percent, as in this example. Certain cases examined in Appendix A, where inflation rates of less than 10 percent are used, can be considered as equally applicable to either estimated or limited values. In cases where very high inflation rates are used, the arguments in the text and Appendix A generally need only minor modification. Where destablishing tendencies occur, constraints on increases in limited values may tend to reduce the size of fluctuations or deviations.

\(^2\) The calculated arithmetic mean would be influenced by the number of sales in each year of the sample. The aggregate mean is a dollar weighted average; it will be influenced by the monetary value of total sales in each year. Since a high number of sales will generally be associated with a high total dollar value, similar fluctuations can be expected in both types of averages. The only qualifying factor stems from inflation--given inflation, fewer sales are necessary in a recent year to produce a given dollar magnitude.
continually requires resources to be devoted to the detection of sale chasing.¹ This is unnecessary, given proper monitoring and safeguards, if sales (or appraisals) are matched with assessor's market values which correspond to the year of sale (or appraisal). Any incentive to chase sales can be eliminated, since this practice could not influence the calculated average assessment/sales ratios and dispersion measures, nor could it increase aids.

The problems caused by the Department's data matching procedures have very serious consequences, whether or not property values are inflating. Given current inflation and the likelihood of its continuance, if assessor's market values do not correspond to the year of sale (or appraisal), little meaningful improvement in the studies is possible.

The Department of Revenue is aware of the problems inherent in the present system of data matching. The reluctance to match sale prices to assessor's market values in the year of sale is due to the belief that a district will lose aids if an assessor improved the level of his assessment during the course of the study. This would be true, given the present approach used to calculate adjusted assessed values. In Chapter III, however, we recommend changes in the adjusted assessed value calculation which will correct this problem.

FINDINGS:

(1) The average assessment/sales ratios and dispersion measures calculated by the Department of Revenue overstate the true levels. Serious biases are caused by the present system of data matching, which matches all sales (or appraisals) in the multi-year sample with the newest assessor's market values. Specifically:

(a) If there is inflation, the average assessment/sales ratios and their dispersion tend to be overstated.

(b) Different rates of inflation will result in different biases in the averages and dispersion measures.

(2) Due to the present system of data matching, the calculated averages are sensitive to the number and volume of sales in each year of the study. An unusually high volume of sales in a given year will cause a "ripple effect" in the means over time. As these sales enter the sample the means will fall; then they will increase until these sales leave the three-year sample, at which time the means will again fall.

¹This task is generally performed by the Property Equalization Section of the Department of Revenue.
(3) As indicated in (1) and (2) above, the calculated averages will overstate the true values by disproportionate amounts and will be subject to unwarranted variation over time. This will distort the allocation of school aids and produce indicators which are not acceptable for use in reassessment efforts.

RECOMMENDATIONS:

(1) All sales (or appraisals) should be matched with assessor's market values in the year of sale (or appraisal). This is the fundamental step which must be taken if any significant improvement is to occur in the sales ratio studies. In addition, this improved data matching should eliminate the potential problem of sale chasing by assessors.
CHAPTER II
SCREENING PROCEDURES

If the information contained in the Certificate of Real Estate Value is to be useful in determining average assessment/sales ratios and dispersion statistics, the incoming certificates must be screened so the sample will contain only transactions between willing, impartial buyers and sellers ("arm's length" transactions). For instance, if sales between relatives, or "sales" of property from one division of a corporation to another division are included, the recorded sales price may not be indicative of that property's value in the market. Including such sales may lead to an erroneous estimate of the average assessment/sales ratios in the community.

Several screening procedures are used to eliminate inappropriate sales. When the Department receives the certificates, each one is manually examined to determine its acceptability. Those observations which survive this manual screening are sorted by county, city or town, and property type. The significant categories are residential, commercial and industrial, apartment, seasonal recreational, improved agricultural land, and unimproved agricultural land. After updating the data from earlier years in the studies, all data are further examined in a series of computerized edit procedures. All updating and subsequent screening are performed for two sets of assessment/sales ratios--one based on limited market values and the other based on estimated market values.

DATA UPDATING

Since the studies are currently based on a multi-year sample using the newest assessor's market values for all observations in the sample, all data must be updated every year. In the 1976 studies all sales (and appraisals when used) from 1976, 1975, 1974, and in some cases 1973 were matched with 1976 assessor's market values. For the 1977 studies the 1975 and 1976 observations were still part of the studies, but now these sales had to be matched with 1977 assessor's market values. Since errors can be introduced in this updating process, all the data, not just the most recent year's observations, are screened annually.

The updating processes differ somewhat between computerized and non-computerized counties. For the non-computerized counties, updating is accomplished through the update report. The update report is a listing of all properties which have at a minimum passed the Department's initial manual screening. For the 1977 study this included essentially all 1975 and 1976 sales or appraisals.

1 Beginning in 1976 commercial and industrial properties were combined into one category labeled "commercial."
used in previous studies and the new 1977 sales or appraisals. In
the update report the property type and sale price are given for
each parcel, along with an identification number, plot and parcel
numbers, and other data. A space on the listing is provided for
the current assessor's market values, which are manually obtained
from county records by field personnel. In the process, the data
receive a screening in the field. These update reports are then
returned to the Department of Revenue, any necessary clerical
changes are made, and the usable data are keypunched in prepara­
tion for two computerized screenings.

In 1976, the following counties were computerized: Anoka, Dakota, Dodge, Hennepin, Itasca, Olmsted, Ramsey, St.
Louis, and Washington. Each computerized county sends to the
Department of Revenue either a computer tape or computer cards
which contain information on all units of property in the county,
both sale and non-sale properties. The Department of Revenue has
a data tape for all properties which are part of its sample. From
the Department's data tape and the county's new data, the new
assessor's market values for sale properties are extracted and
merged with sale prices to form individual assessment/sales ratios.

For the smaller computerized counties, an update listing
is then produced which is examined at the Department of Revenue.
For the largest counties, however, there are too many observations
to produce and use an update listing. In 1976, these were
Hennepin, Anoka, Ramsey, St. Louis, Washington, and Dakota
counties; together, they accounted for nearly half the observations
in the statewide sample.

COMPUTERIZED SCREENING PROCEDURES

The remaining steps are identical for computerized and
non-computerized counties. The data are run through two com­
puterized screening procedures. The first computer edit is the
"possible errors run," primarily designed to detect keypunch errors
and errors which may have occurred in the updating phase. The
program examines the data for:

(1) Sales which are too old: For the 1977 run, any sales
from 1974 or earlier which somehow were not eliminated
from the sample were deleted at this stage.

(2) Duplicates: If the same sale is accidentally included more
than once, the duplicates are eliminated.

(3) Nonexistent property types, and wrong or nonexistent
county or town codes: These are probably key­
punching errors.
Improper acreage or sales that are too low in value: Sales of farms with less than 20 acres and all observations with prices less than $1,000 receive further examination. The low acreage may indicate a keypunch error, or the property may be in the wrong category. A sale price less than $1,000 for any property type may indicate an error in keypunching. If the keypunching was correct, all sales less than $1,000 are deleted.

Zero ratios: If a ratio has a zero value, either the sale price or assessor's values were not keypunched, or identification information is incorrect. For computerized counties, incorrect identifier information would make it impossible to match a given sale price with the new assessor's market values from the county's data file; hence a zero ratio would appear.

Having passed the possible errors run, the data are further checked using the "extreme ratio listing." This procedure is designed to detect individual assessment/sales ratios which differ substantially from the average ratios for each property type, by county. These "outliers" can then be re-examined to determine if they represent valid observations. An outlier may be due to poor assessor performance; however it may represent a keypunch error or a non-arm's length transaction which was not detected earlier.

The extreme ratio listing selects the median ratio for each property type in each county and then prints out all individual assessment/sales ratios which lie 25 points above or below this median. In re-examining these outliers, the first step is to re-check the Certificate of Real Estate Value; then if necessary the values are checked by telephone with the county assessor. If there are many outliers, they may be checked by field staff.

DEFICIENCIES IN SCREENING PROCEDURES

The present system of data matching leads to difficulties which significantly reduce the effectiveness of the present computerized screening procedures. The updating phase, which is necessary given the present system of data matching, could be eliminated if the assessor's market values used in the studies corresponded to the year of sale (or appraisal), as recommended in Chapter I. This would enable more time and resources to be devoted to actual data screening, and it would considerably decrease the volume of data to be screened. Since errors can occur in the updating phase, it is currently necessary to screen all data in the study.

1Another possibility is that the newest assessor's market values reflect improvements in the property which have occurred since the sale.
recommended form of data matching, only the newest year's data would require screening, reducing the volume of data to be screened to roughly one-third its current level.

The present system of data matching directly reduces the effectiveness of the extreme ratio listing. Since all sales are currently matched with the newest assessor's values, during inflationary periods old sales tend to have high ratios while new sales have lower ratios. As a result, old sales predominate among the high outliers merely because they are old. New sales predominate among the low outliers merely because they are new. Thus many of the sales in these outlier regions are reliable, and attention may be drawn away from truly deviant observations.

New sales entering the sample unquestionably deserve the most attention. Older sales have already survived one or two complete screenings. The major concern with older sales is finding those errors which were introduced in the latest updating. However, due to the Department's data matching system, which tends to inflate the ratios for older sales, many new ratios which are abnormally high may not even appear among the high outliers. Furthermore, if they do appear, they may not be examined. Due to time limitations not all outliers are checked, and the fraction of those which are checked varies from county to county.

For some computerized counties obtaining data from the county data files during the updating phase has been difficult, and in other cases the tapes have arrived too late to be screened according to standard procedures. Occasionally, the usual edits have not been run sequentially, thus reducing their effectiveness. In the 1976 study for residential property in one computerized county, 229 observations appeared on the extreme ratio edit with zero ratios. This would not occur if the proper screening procedures were followed sequentially. When the data are initially selected from the county tape, observations which cannot be matched are identified and should be corrected or deleted at this stage. These errors would again be observed in the possible errors run, which prints all observations with zero ratios.

Department staff also indicated that some edit procedures may have been completely omitted, although no specific omissions were identified.

When the effectiveness of early screening procedures is impaired, the burden is placed upon the extreme ratio edit. As previously stated, the extreme ratio edit is not effective due to the present data matching system and because not all outliers are actually checked. Another difficulty, although minor, is that zero ratios on this edit may have the additional consequence of artificially lowering the median. The median is the "middle" ratio as each individual ratio is ranked from smallest to largest. If the program uses these zero ratios in determining the median, the out-
FINDINGS:

(1) The present system of data matching, where all sales are matched with the newest assessor's market values, impairs the quality of the screening process.

(a) Since all observations must be reconstructed every year, update reports (or their equivalent in computerized counties) must be performed annually. This decreases the time available to edit the data and perform the actual studies.

(b) Because errors can be introduced in the updating phase, the present system necessitates annual screening of all data, increasing the burden on the available staff.

(c) The present data matching system impairs the effectiveness of the extreme ratio listing. Old sales predominate in the high range merely because they are old, while new sales predominate in the low range merely because they are new.

(2) The quality of the screening is not consistent, varying among counties.

(a) While all ratios 25 points above and below the median appear on the extreme ratio listing, not all are checked due to time and personnel constraints. The percentage of the extreme ratios checked differs among counties.

(b) Problems which have impaired the screening of data have arisen in computerized counties. The tapes from some counties have arrived late, and difficulties have occurred in obtaining information from many of these tapes. As a result some of the computer screenings have either been omitted entirely or the screenings have not been run sequentially, which reduces their effectiveness.

1 It was not possible to conclude positively that zero ratios are incorporated in the calculation of this median. However, if the zero ratios are ignored in calculating the median, it would then be peculiar to include these ratios in the outliers, as is currently the case. Second, it is unlikely that the computer programs were designed to ignore zero ratios, since zero ratios would normally never appear on this listing.
RECOMMENDATIONS:

(1) As recommended in Chapter I, all observations should be matched with assessor's market values in the year of sale. This will permit the following changes which improve the quality of the screening procedures:

(a) Only the newest year's data would need to be screened. The amount of data to be scrutinized annually will decrease to approximately one-third its current level. Data from previous years which remain in the study will have previously passed through the screening procedures and can be accepted as valid observations.

(b) The use of computerized and manual update reports can be eliminated. This would permit any problems with computerized counties to be completely circumvented, leading to more uniformity in data quality from county to county. Eliminating the update reports would also permit data editing to begin earlier and to be performed more intensively.

(2) Given adoption of the data matching recommendation in Chapter I, the Department of Revenue should consider reducing the 25 point cutoffs used in the extreme ratio runs. The reasons are:

(a) As mentioned in Chapter I, the present system of data matching artificially increases the dispersion. Thus present cutoffs may be too wide.

(b) With sales matched with assessor's market values in the year of sale, the data requiring screening will be roughly one-third the current amount. Therefore cutoffs can be reduced, still permitting intensive re-examination. The interval selected should be as small as possible, subject to the constraint that all outliers are examined.
CHAPTER III

THE ADJUSTED ASSESSED VALUE CALCULATION

As noted in the introduction to this report, the adjusted assessed value is used as an indicator of a community's property wealth and is a key factor in determining the amount of school aids a community receives. Adjusted assessed value is defined as the assessed value that would be obtained if the assessor were valuing all property at its full value in the market. The school aid formula is designed so that the higher the adjusted assessed value, the more funds must be raised from local taxes. Communities with comparable adjusted assessed values raise equal amounts of local tax revenues for school funding.

This chapter first briefly describes the computerized stages which lead to the calculation of the adjusted assessed values. Next, the methodology currently used to calculate these values is demonstrated. This approach is then compared to a procedure which will yield more acceptable estimates than are obtained with the present procedure.

After computing the average assessment/sales ratios and measures of dispersion, the Department of Revenue publishes this information in its Real Estate Assessment/Sales Ratio Study. This information is also stored for further use in the computerized stages of school aid determination. The final output of these computerized stages is the "final tape verification listing." This listing is formed by merging the "Abstract of Assessment tape file," which contains the total limited market value for each property type, by town and county within each school district, with the aggregate assessment/sales ratio for each property type. In cases where the sample size is too small to justify use of the city or town ratio, either a county wide ratio is substituted, or a ratio from a similar property type is manually matched with the pertinent information from the Abstract of Assessment file.1 Once the limited market values have been matched with aggregate assessment/sales ratios, each total limited value is divided by the assessment/sales ratio, producing indicated market values. These steps appear on the final tape verification listing. The adjusted assessed values are then manually calculated from this information.

THE ADJUSTED ASSESSED VALUE CALCULATION

In this section, deficiencies in the Department's current procedure for calculating adjusted assessed values are illustrated by

1 Problems with computer programs used in this matching procedure and with the decision rules which determine when county-wide ratios are used are discussed in Chapter IV.
examples. We first demonstrate how the Department of Revenue calculates adjusted assessed values and then present a procedure that yields more accurate results.

These examples will incorporate two property types, residential and commercial. Assume, for example, that the total limited market value of residential property is $4,000,000 and its aggregate assessment/sales ratio is 80 percent. If the classification ratio for the property is assumed to be 22 percent, the assessed value is $880,000 ($4,000,000 x 22% = $880,000). We also assume the total limited market value for commercial property is $1,000,000, its aggregate assessment/sales ratio is 50 percent, and its classification ratio is 43 percent. The assessed value for commercial property is thus $430,000 ($1,000,000 x 43% = $430,000).

The Department first selects each limited market value and divides it by the assessment/sales ratio for its property category, obtaining indicated market values. The indicated market value is the limited market value that would be recorded if the assessor were valuing properties at their true market value. These results appear in Table 1. Dividing the sum of the limited market values

<table>
<thead>
<tr>
<th>Limited Market Value</th>
<th>Assessment/Sales Ratio</th>
<th>Indicated Market Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Residential</td>
<td>$4,000,000</td>
<td>80%</td>
</tr>
<tr>
<td>Commercial</td>
<td>$1,000,000</td>
<td>50%</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>$5,000,000</strong></td>
<td><strong>71.4%</strong></td>
</tr>
</tbody>
</table>

1 Each property type has a different classification ratio which determines the fraction of limited market value which can be considered for taxation purposes. The assessed value is obtained by multiplying the limited values by the classification ratio for that property type. Non-homestead residential property is "assessed" at 40 percent of limited market value. Homesteaded residential property has a split classification ratio of 22 percent for limited value below $15,000 and 36 percent for limited value above $15,000. In the above example it is assumed that all residential property is homesteaded, and each unit has a limited market value of less than $15,000. Otherwise complications caused by the split classification system would be introduced in the example.
by the sum of the indicated market values yields an average of 71.4 percent. This is a weighted average, lying between the average ratio for residential, 80 percent, and the average ratio for commercial, 50 percent.

To calculate adjusted assessed values for this example, the Department would manually divide each total assessed value by the weighted average ratio\(^1\) derived above (71.4 percent) rather than dividing the residential assessed value by the assessment/sales ratio for residential property and the commercial assessed value by the commercial assessment/sales ratio. Table 2 shows these values, following the Department's procedure.

### TABLE 2

<table>
<thead>
<tr>
<th></th>
<th>Assessed Value</th>
<th>Assessment/Sales Ratio</th>
<th>Adjusted Assessed Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Residential</td>
<td>$880,000</td>
<td>71.4%</td>
<td>$1,232,493.00</td>
</tr>
<tr>
<td>Commercial</td>
<td>$430,000</td>
<td>71.4%</td>
<td>$602,240.90</td>
</tr>
<tr>
<td>Total</td>
<td>$1,310,000</td>
<td>71.4%</td>
<td>$1,834,733.90</td>
</tr>
</tbody>
</table>

The resulting adjusted assessed values for each property type and the total adjusted assessed value are inaccurate. Residential property has an assessment/sales ratio of 80 percent, and commercial property has a ratio of 50 percent. Therefore each adjusted assessed value derived by using the weighted ratio, 71.4 percent, is inaccurate. The total is also inaccurate, as can be seen by comparing the $1,834,733.90 figure with the result obtained in Table 3.

In Table 3 each assessed value is divided by the assessment/sales ratio for its property type. For instance, the assessed value of residential property is $880,000; however, on average the assessor is valuing property at 80 percent of selling price. If property were valued at 100 percent of current market value, there would be $1,100,000 of residential assessed value ($880,000 ÷ 80\% = $1,100,000).

\(^1\)In practice the assessed values are summed and the total assessed value is divided by the weighted average ratio. The procedures are equivalent, but the description in the text provides more insight into the implications of the present approach.
The same procedure yields an adjusted assessed value of $860,000 for commercial property. Adding the two, the total adjusted assessed value is $1,960,000. This differs from the results obtained under current Department of Revenue practice as illustrated in Table 2.

### THE SOURCE OF THE ERROR IN ADJUSTED ASSESSED VALUES

The preceding examples illustrate that the procedure currently used by the Department of Revenue to calculate adjusted assessed value generally produces inaccurate results. The adjusted assessed value obtained using the Department of Revenue methodology is incorrect because the assessment/sales ratio used in the calculation is implicitly weighted by indicated market values, which are calculated before the classification ratios are applied. Therefore the calculated adjusted assessed value does not properly reflect legislative intent to spread the property tax burden by use of classification ratios. In contrast, the procedure recommended in Table 3 implicitly uses weights which reflect the classification ratios.

This can be demonstrated by examining the ratio used in the Department’s procedure to calculate the adjusted assessed value. In Table 1 the sum of the limited market values is divided by the sum of indicated market values, producing a weighted-average ratio of 71.4 percent. To estimate the total adjusted assessed value for a district, each assessed value is divided by 71.4 percent and the resulting values are summed. Alternatively, the assessed values for all property types are summed and the total, $1,310,000, is divided by 71.4 percent.

The ratio used, 71.4 percent, is an assessment/sales ratio weighted by the fraction of indicated market values in each category. To illustrate, in Table 1 the total indicated market value is $7,000,000. Of this total, residential property comprises $5,000,000 or 71.4 percent. Commercial property accounts for $2,000,000 or 28.6 percent of the total indicated market value. If the aggregate assessment/sales ratio for residential property is multiplied by 71.4...
percent, and this is added to the commercial aggregate ratio multiplied by 28.6 percent, the result is the ratio used in Table 2:

\[(1) \ (80\% \times 71.4\%) \ + \ (50\% \times 28.6\%) \ = \ 71.4\% .\]

In contrast, the procedure suggested in Table 3 uses weights which reflect the classification ratios. The correct total adjusted assessed value, $1,960,000, can be derived by dividing the total assessed value, $1,310,000, by 66.8%. This assessment/sales ratio is weighted by the fraction of total adjusted assessed value in each category. The total adjusted assessed values are calculated from the assessed values, which incorporate the classification ratios. Using the information in Table 3, 56 percent ($1,100,000 \div \$1,960,000) of the total adjusted assessed value is in residential property, while 44 percent is in commercial property. Using these weights, the ratio 66.8 percent is derived:

\[(2) \ (80\% \times 56\%) \ + \ (50\% \times 44\%) \ = \ 66.8\%.\]

Thus, the system currently used to calculate adjusted assessed values does not produce accurate estimates. The estimates do not accurately indicate the true taxable market value of a district. The source of the problem lies in the weighting system used; the weights do not reflect the legislature's decision to spread the property tax burden through the use of classification ratios.¹

The values calculated using the present system may either underestimate or overestimate the actual values. Only under special and unrealistic conditions will the current method yield accurate results. The Department's method will yield accurate estimates only if all classification ratios, or if all average assessment/sales ratios, are the same. This can be seen by referring to calculations (1) and (2) above. These calculations differ only in the weights used. In calculation (1), which illustrates Revenue's current procedures, the percentages of total indicated market value in each category, 71.4% and 28.6%, are used as weights. In calculation (2), the percentages of adjusted assessed value in each category, 56% and 44%, are used as weights. If all classification ratios are identical, the weights in (1) and (2) would be the same, and the calculations would be identical. Revenue's procedure would also yield an accurate result if all average assessment/sales ratios were identical. This is true because in this case the calculation will always yield the common average assessment/sales ratio.

¹In fairness, it should be noted that the data initially collected by the Department were not sufficiently detailed to permit implementation of this correct method of calculating adjusted assessed values. However, in recent years the suggested improvements have been feasible.
Neither of the conditions sufficient for accurate results is found in practice. The legislature has established separate classification ratios for each property type, and the findings in Chapter IV in this report, as well as additional research by the Program Evaluation Division, demonstrate that average assessment/sales ratios differ by property type.

AN ADDITIONAL IMPROVEMENT: THE USE OF AVERAGE TOTAL ASSESSED VALUE

To calculate adjusted assessed values which adequately reflect taxable property wealth, the total assessed value for each property type by town and county within each school district should be divided by the aggregate assessment/sales ratio for that property type. The adjusted assessed values can then be summed to obtain the total adjusted assessed value for the school district. This is the procedure suggested in the previous section.

Besides this basic change, an additional refinement should be implemented. Currently, only the total assessed value from the final year of the study is used in the adjusted assessed value calculation. The adjusted assessed value should be calculated by taking a multi-year average of total assessed values and dividing this by the aggregate assessment/sales ratio. Specifically, if a three year sample is used to calculate the assessment/sales ratios, then the recommended procedure is to use a three year sales-value-weighted average of the total assessed value, by town and property type within the district, and to divide this by the appropriate aggregate assessment/sales ratio. For best results, the same type of average must be used in the numerator (the total assessed values) and the denominator (the assessment/sales ratio). The aggregate assessment sales ratio actually is a sales-value-weighted average; therefore a sales-value-weighted average is necessary in the numerator. Two different forms of averages should not be used.

To illustrate the recommended procedure, data from 1975, 1976, and 1977 would be used to calculate the 1977 adjusted assessed value by town and property type within a district. The total assessed value in 1975 would be multiplied by the fraction of total sales value from 1975, 1976, and 1977 that occurred in 1975; the total assessed value in 1976 would be multiplied by the fraction of sales value that occurred in 1976; and the total assessed value in

In addition to the findings which appear in Chapter IV concerning commercial, industrial, and agricultural properties, Mann-whitney tests were performed to determine whether commercial and industrial properties can be assumed to have assessment/sales ratios similar to residential property. The tests showed frequent differences between the average assessment/sales ratios for residential property and those for commercial and industrial property.
1977 would be multiplied by the fraction of sales value that occurred in 1977. These figures would be summed and then divided by the aggregate assessment/sales ratio, which is based on 1975, 1976, and 1977 sales (or appraisals).

The purpose of this procedure is to produce an indicator of adjusted assessed value which has the properties of an average. When property wealth changes, due to appreciation, depreciation, improvements, or new construction, the changes will gradually affect adjusted assessed value. This will increase the stability of school aids.

In addition, given proper data matching, when a single year's total assessed value is used in combination with a three year aggregate assessment/sales ratio, if an assessor improves his performance during the course of the study the district will be penalized. If the assessment level increases aids would fall. Our recommendation concerning the use of a sales-value-weighted average of total assessed values will correct this problem. In the appendix, numerical examples are used to demonstrate the recommended procedure. A more formal description of the properties of the procedure is available from the Program Evaluation Division.

FINDINGS:

(1) The procedure now used for calculating adjusted assessed values almost invariably yields inaccurate results. The Department's method will yield an accurate total only if all classification ratios are the same, or if the average assessment/sales ratios are identical for all property types. Neither of these conditions is true in practice.

(2) The present procedure for calculating adjusted assessed values is inconsistent with legislative intent to spread the property tax burden through use of classification ratios.

RECOMMENDATION:

(1) Given adoption of the data matching recommendation in Chapter I, to calculate adjusted assessed values a sales-value-weighted average of total assessed value should be divided by the appropriate aggregate assessment/sales ratio for each property type and municipality. The number of years of total assessed values used in the calculation should match the number of years used in calculating the aggregate assessment/sales ratio.
CHAPTER IV
FURTHER DATA ISSUES

This chapter contains five sections. The first section deals with the general need for personnel capable of designing and implementing statistical testing procedures. The second section examines the use of appraisals to expand the sample for several property types. The appraisals are used to form individual assessment/appraisal ratios, which are added to the sample of assessment/sales ratios when there are few sales. The next section pertains to the decision to combine commercial and industrial property into a single category beginning with the 1976 studies. If commercial and industrial property are not comparably assessed, combining the two has detrimental effects upon aid allocation and undesirable implications for the use of appraisals. The fourth section discusses the agricultural sample, where the problems are somewhat similar in nature to commercial and industrial issues. The final section examines deficiencies in computer programs which are used to select aggregate assessment/sales ratios for eventual calculation of adjusted assessed values. The adequacy of guidelines for selecting city or town aggregate ratios versus county ratios for use in these calculations is also addressed in this section.

THE GENERAL NEED FOR STATISTICAL ANALYSIS

Several sections of this report employ statistical procedures to test the admissibility of certain data for the studies. The recommendations associated with these sections advise using statistical tests when the validity of data is questionable. To implement these recommendations and to develop additional procedures where necessary, the Department will require staff competent in advanced statistical techniques. In light of these substantial needs, we make the following general recommendation:

RECOMMENDATION:

(1) The Department of Revenue should hire additional personnel to implement statistical testing procedures relating to the design and uses of assessment/sales ratio studies.

THE USE OF APPRAISALS

In some cases where the sample size has been deemed insufficient, the Department of Revenue has enlarged the sample by using appraisals. These appraisals are then used to form individual assessment/appraisal ratios which are added to the sample of assessment/sales ratios.
If these assessment/appraisal ratios are comparable to assessment/sales ratios, then enlarging the sample through use of appraisals will improve the estimate of the mean sales ratios and improve the quality of the studies. If the individual assessment/appraisal ratios are not comparable to individual assessment/sales ratios, then the resulting estimates of the means will be distorted. The averages calculated from these data will not be indicative of the true assessment level, and the assessment/sales ratio studies and school aid allocations will be impaired. In this case, better estimates are obtained if the appraisals are not included.

Appraisals are frequently used for the commercial and industrial category, and in the past they have been used in the agricultural, apartment, and residential samples. To determine whether appraisals may have been inappropriately used, we performed Mann-Whitney tests on county-wide data for commercial property. This category was selected because appraisals are currently used in this category and because there were sufficient sales and appraisals to conduct the test in most counties. Industrial properties could not be tested because of insufficient sales in our sample.

The Mann-Whitney test is a standard statistical test which can be used to determine whether two data sets (in this case a set of individual assessment/sales ratios and a set of individual assessment/appraisal ratios) can be safely combined. The test produces a probability which may be loosely interpreted as the probability that the two data sets are comparable. For instance, if the calculated probability is 90 percent, we are 90 percent sure that the two subsets of ratios are comparable, and that by combining the two sets into one larger sample we will obtain a better estimate of the average sales ratio. If we perform the tests and find a probability of 10 percent, we have only a 10 percent probability that the subsets are comparable, and hence it is very likely that using appraisals would only lead to a worse estimate of the average assessment/sales ratios.

The data used in these tests were observations from 1974, 1975, and 1976 commercial sales and appraisals. The sale price (or appraisal) was matched with the assessor's market value in the year of sale (or appraisal). Each of the counties and the associated probabilities are listed in Table 4.

When interpreting the probabilities in this table, several factors should be remembered. First, a low probability of comparability is not a strong indication that the appraisals were poor. The test yields a probability that assessment/sales ratios were comparable to assessment/appraisal ratios. If the probability is low, it may...

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1 The term "commercial property" used here is not synonymous with the current Department of Revenue category which includes both commercial and industrial property. We have eliminated industrial properties from the sample, maintaining consistency with the recommendations in this chapter.
TABLE 4
Probabilities of Comparability Between Assessment/Sales Ratio Data and Assessment/Appraisal Ratio Data: Commercial Property

<table>
<thead>
<tr>
<th>County</th>
<th>Probability</th>
<th>County</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aitkin</td>
<td>23.64%</td>
<td>Fillmore</td>
<td>37.03%</td>
</tr>
<tr>
<td>Anoka</td>
<td>5.27%</td>
<td>Freeborn</td>
<td>*</td>
</tr>
<tr>
<td>Becker</td>
<td>68.55%</td>
<td>Goodhue</td>
<td>16.50%</td>
</tr>
<tr>
<td>Beltrami</td>
<td>3.89%</td>
<td>Grant</td>
<td>*</td>
</tr>
<tr>
<td>Benton</td>
<td>42.39%</td>
<td>Hennepin</td>
<td>.57%</td>
</tr>
<tr>
<td>Big Stone</td>
<td>61.04%</td>
<td>Houston</td>
<td>27.23%</td>
</tr>
<tr>
<td>Blue Earth</td>
<td>87.46%</td>
<td>Hubbard</td>
<td>49.69%</td>
</tr>
<tr>
<td>Brown</td>
<td>18.46%</td>
<td>Isanti</td>
<td>*</td>
</tr>
<tr>
<td>Carlton</td>
<td>30.63%</td>
<td>Itasca</td>
<td>20.19%</td>
</tr>
<tr>
<td>Carver</td>
<td>32.20%</td>
<td>Jackson</td>
<td>*</td>
</tr>
<tr>
<td>Cass</td>
<td>28.23%</td>
<td>Kanabec</td>
<td>22.20%</td>
</tr>
<tr>
<td>Chippewa</td>
<td>*</td>
<td>Kandiyohi</td>
<td>*</td>
</tr>
<tr>
<td>Chisago</td>
<td>13.45%</td>
<td>Kittson</td>
<td>*</td>
</tr>
<tr>
<td>Clay</td>
<td>8.26%</td>
<td>Koochiching</td>
<td>34.02%</td>
</tr>
<tr>
<td>Clearwater</td>
<td>*</td>
<td>LacQui Parle</td>
<td>95.39%</td>
</tr>
<tr>
<td>Cook</td>
<td>20.59%</td>
<td>Lake</td>
<td>56.28%</td>
</tr>
<tr>
<td>Cottonwood</td>
<td>*</td>
<td>Lake of the Woods</td>
<td>40.00%</td>
</tr>
<tr>
<td>Crow Wing</td>
<td>94.38%</td>
<td>LeSueur</td>
<td>88.00%</td>
</tr>
<tr>
<td>Dakota</td>
<td>*</td>
<td>Lincoln</td>
<td>*</td>
</tr>
<tr>
<td>Dodge</td>
<td>5.40%</td>
<td>Lyon</td>
<td>62.44%</td>
</tr>
<tr>
<td>Douglas</td>
<td>7.36%</td>
<td>McLeod</td>
<td>3.45%</td>
</tr>
<tr>
<td>Faribault</td>
<td>71.99%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Due to insufficient sample size this county could not be analysed.
<table>
<thead>
<tr>
<th>County</th>
<th>Probability</th>
<th>County</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mahnomen</td>
<td>5.60%</td>
<td>Scott</td>
<td>73.21%</td>
</tr>
<tr>
<td>Marshall</td>
<td>1.76%</td>
<td>Sherburne</td>
<td>16.49%</td>
</tr>
<tr>
<td>Martin</td>
<td>19.73%</td>
<td>Sibley</td>
<td>100.00%</td>
</tr>
<tr>
<td>Meeker</td>
<td></td>
<td>Stearns</td>
<td>2.02%</td>
</tr>
<tr>
<td>Mille Lacs</td>
<td>81.41%</td>
<td>Steele</td>
<td>46.85%</td>
</tr>
<tr>
<td>Morrison</td>
<td>10.23%</td>
<td>Stevens</td>
<td>15.73%</td>
</tr>
<tr>
<td>Mower</td>
<td>65.20%</td>
<td>Swift</td>
<td>10.00%</td>
</tr>
<tr>
<td>Murray</td>
<td></td>
<td>Todd</td>
<td>49.23%</td>
</tr>
<tr>
<td>Nicollet</td>
<td>34.91%</td>
<td>Traverse</td>
<td></td>
</tr>
<tr>
<td>Nobles</td>
<td>13.17%</td>
<td>Wabasha</td>
<td>91.31%</td>
</tr>
<tr>
<td>Norman</td>
<td>43.34%</td>
<td>Wadena</td>
<td>61.04%</td>
</tr>
<tr>
<td>Olmsted</td>
<td></td>
<td>Waseca</td>
<td>45.02%</td>
</tr>
<tr>
<td>Otter Tail</td>
<td>54.51%</td>
<td>Washington</td>
<td>84.38%</td>
</tr>
<tr>
<td>Pennington</td>
<td>37.98%</td>
<td>Watonwan</td>
<td>60.38%</td>
</tr>
<tr>
<td>Pine</td>
<td>4.26%</td>
<td>Wilkin</td>
<td>80.65%</td>
</tr>
<tr>
<td>Pipestone</td>
<td></td>
<td>Winona</td>
<td>35.80%</td>
</tr>
<tr>
<td>Polk</td>
<td>20.97%</td>
<td>Wright</td>
<td>98.43%</td>
</tr>
<tr>
<td>Pope</td>
<td>52.74%</td>
<td>Yellow Medicine</td>
<td>17.42%</td>
</tr>
<tr>
<td>Ramsey</td>
<td>98.56%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Red Lake</td>
<td>25.68%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Redwood</td>
<td>100.00%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Renville</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rice</td>
<td>57.96%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rock</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Roseau</td>
<td>29.63%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>St. Louis</td>
<td>9.30%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Due to insufficient sample size this county could not be analyzed.
mean the appraisals were not comparable to sales; it may also indicate inconsistent assessment performance.¹

Second, the ability of the test to determine if there is a difference between the two types of ratios depends upon two elements—the number of sales and appraisals, and the apparent difference between the averages for the two types of ratios. If there is only one sale and one appraisal, it is impossible to safely conclude that the ratios are not comparable, regardless of how divergent the individual ratios may be. If, however, the sample is large, it is often possible to conclude that the ratios are not comparable, even if the difference between the average assessment/sales ratio and the average assessment/appraisal ratio is small. Therefore, it is not permissible to assume that adjusted assessed values, and hence school aids, were more distorted in counties with lower probabilities. What can be concluded is this: if the probability of comparability is low, it is very likely that the average assessment/sales ratios and consequently school aids were distorted to some degree.

FINDING:

(1) Statistical tests indicate that in some cases the use of appraisals in past studies was detrimental, resulting in poorer estimates of average assessment/sales ratios. In 18 counties the sample size was insufficient to generate a test result. Of the 69 testable counties, nearly one-third (21 counties) had a probability of being comparable of less than 20 percent. In 11 of these 21 counties, the probability of being comparable was less than 10 percent.

The following recommendations are contingent upon the adoption of recommendation 1 in Chapter I concerning data matching. The recommendations below should not be followed if the present system of data matching is continued. Under such circumstances, the use of statistical testing may be detrimental. Given the present system of data matching, adoption of the first recommendation below would increase the variability over time of the calculated averages. (See Section 7 of Appendix A).

¹If further statistical testing revealed that the assessor tends to underassess high value property relative to low value property, there might be a need to stratify the sample by value range to improve the accuracy of the adjusted assessed value estimate. The decision would depend upon the cost of performing this procedure versus the expected gain in precision.
RECOMMENDATIONS:

(1) The Department of Revenue should develop and implement a test procedure to determine the advisability of using a particular set of appraisals. An alternative, the use of a fourth year of sales, should also be tested. A set of guidelines for testing appears in the recommendations for the next section.

(2) The Department of Revenue should develop a rigorous procedure to determine where sample size should be increased, and by how much the sample should be expanded. Currently the decision to use additional appraisals or sales lacks a rigorous, consistent basis. The procedure developed should carefully weigh the cost of expanding the sample, either through appraisals or a fourth year of sales, against the gain in precision.¹

COMBINING COMMERCIAL AND INDUSTRIAL PROPERTY

Prior to the 1976 sales ratio study, data for commercial and industrial property were maintained separately, and average assessment/sales ratios were calculated for each category. Beginning with the 1976 study, however, the two property types were combined and only one set of average assessment/sales ratios was calculated for the combined category.

If the two property types are comparably assessed, no impairment results from combining them into a single category for the study. However, if the property types are not comparably assessed, the following problems will occur:²

¹ A foundation for determining adequate sample size is to observe confidence intervals for the various property types in each municipality. While not entirely adequate, since the aggregate mean is the measure used extensively in school aids, the easiest confidence interval to use is the interval for the arithmetic mean. A discussion of appropriate procedures is available from the Program Evaluation Division. In determining the geographic areas and property types to concentrate efforts, the goals and tradeoffs must be kept in mind. If we wish to use a sales ratio study for reassessment purposes, the areas where additional sales or appraisals should be used could differ from the samples to be expanded if the goal were to increase accuracy in school aid allocation.

² The following problems are demonstrated with examples in Appendix C.
The adjusted assessed values will be distorted. The direction of the error cannot be predicted without detailed information. The direction will depend upon:

(a) the relative magnitude of total industrial assessed value vs. total commercial assessed value;

(b) the relative proportions of industrial versus commercial assessor's market value in the sales ratio samples; and

(c) whether a separate aggregate sales ratio for industrial property would be greater or less than the separate commercial aggregate ratio.

The calculated adjusted assessed value will fluctuate over time, even if actual property wealth is unchanging. This follows from (b) above. For instance, in a given year if an unusual amount of commercial property sells, this will alter the relative proportion of commercial to industrial assessor's market value in the sales ratio sample, which in turn will alter the calculated average assessment/sales ratio, the adjusted assessed value, and the aids received by the district.

The use of appraisals can alter the aid allocation. It is immaterial whether the proportion of commercial to industrial assessor's market value in the sales ratio sample is altered by an unusual number of sales in a given category or by the use of appraisals. Even good appraisals can cause destabilizing effects upon aid distribution.

To determine whether combining commercial and industrial properties is detrimental in practice, Mann-Whitney tests were performed on commercial and industrial properties, by county, to determine the probability that they are comparably assessed. The data used were matched so that assessor's market values correspond to the year of sale (or appraisal). The results appear in Table 5.

Due to the small sample, 21 counties could not be tested. Of the 66 remaining, 35 percent had less than a 20 percent probability of being comparable. Several counties with considerable commercial and industrial wealth had very low probabilities.

These results should not be construed as conclusive proof that distortions exist for particular counties. The commercial sample used in the above tests contained both sales and appraisals, while the industrial sample was almost entirely composed of appraisals. As indicated in the previous section, in some cases it is questionable whether commercial appraisals should be used. Similar caution should be used with industrial appraisals. Due to sample
TABLE 5
Probabilities of Comparability:
Commercial and Industrial Data

<table>
<thead>
<tr>
<th>County</th>
<th>Probability</th>
<th>County</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aitkin</td>
<td>1.34%</td>
<td>Fillmore</td>
<td>7.40%</td>
</tr>
<tr>
<td>Anoka</td>
<td>72.47%</td>
<td>Freeborn</td>
<td>82.54%</td>
</tr>
<tr>
<td>Becker</td>
<td>58.62%</td>
<td>Goodhue</td>
<td>28.39%</td>
</tr>
<tr>
<td>Beltrami</td>
<td>52.54%</td>
<td>Grant</td>
<td>*</td>
</tr>
<tr>
<td>Benton</td>
<td>95.35%</td>
<td>Hennepin</td>
<td>.19%</td>
</tr>
<tr>
<td>Big Stone</td>
<td>74.49%</td>
<td>Houston</td>
<td>10.20%</td>
</tr>
<tr>
<td>Blue Earth</td>
<td>33.34%</td>
<td>Hubbard</td>
<td>*</td>
</tr>
<tr>
<td>Brown</td>
<td>40.70%</td>
<td>Isanti</td>
<td>*</td>
</tr>
<tr>
<td>Carlton</td>
<td>10.00%</td>
<td>Itasca</td>
<td>10.00%</td>
</tr>
<tr>
<td>Carver</td>
<td>*</td>
<td>Jackson</td>
<td>15.85%</td>
</tr>
<tr>
<td>Cass</td>
<td>*</td>
<td>Kanabec</td>
<td>23.86%</td>
</tr>
<tr>
<td>Chippewa</td>
<td>*</td>
<td>Kandiyohi</td>
<td>85.30%</td>
</tr>
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<td>Chisago</td>
<td>91.64%</td>
<td>Kittson</td>
<td>82.73%</td>
</tr>
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<td>Clay</td>
<td>64.04%</td>
<td>Koochiching</td>
<td>32.18%</td>
</tr>
<tr>
<td>Clearwater</td>
<td>*</td>
<td>LacQui Parle</td>
<td>43.38%</td>
</tr>
<tr>
<td>Cook</td>
<td>*</td>
<td>Lake</td>
<td>*</td>
</tr>
<tr>
<td>Cottonwood</td>
<td>16.15%</td>
<td>Lake of the Woods</td>
<td>*</td>
</tr>
<tr>
<td>Crow Wing</td>
<td>.96%</td>
<td>LeSueur</td>
<td>62.00%</td>
</tr>
<tr>
<td>Dakota</td>
<td>59.65%</td>
<td>Lincoln</td>
<td>14.03%</td>
</tr>
<tr>
<td>Dodge</td>
<td>84.74%</td>
<td>Lyon</td>
<td>.37%</td>
</tr>
<tr>
<td>Douglas</td>
<td>50.24%</td>
<td>McLeod</td>
<td>28.02%</td>
</tr>
<tr>
<td>Faribault</td>
<td>34.84%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Due to insufficient sample size these counties could not be tested.
<table>
<thead>
<tr>
<th>County</th>
<th>Probability</th>
<th>County</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mahnomen</td>
<td>*</td>
<td>Scott</td>
<td>41.01%</td>
</tr>
<tr>
<td>Marshall</td>
<td>*</td>
<td>Sherburne</td>
<td>91.36%</td>
</tr>
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<td>Martin</td>
<td>62.77%</td>
<td>Sibley</td>
<td>29.59%</td>
</tr>
<tr>
<td>Meeker</td>
<td>13.71%</td>
<td>Stearns</td>
<td>3.54%</td>
</tr>
<tr>
<td>Mille Lacs</td>
<td>3.67%</td>
<td>Steele</td>
<td>40.72%</td>
</tr>
<tr>
<td>Morrison</td>
<td>2.11%</td>
<td>Stevens</td>
<td>37.98%</td>
</tr>
<tr>
<td>Mower</td>
<td>6.43%</td>
<td>Swift</td>
<td>23.09%</td>
</tr>
<tr>
<td>Murray</td>
<td>*</td>
<td>Todd</td>
<td>19.36%</td>
</tr>
<tr>
<td>Nicollet</td>
<td>6.76%</td>
<td>Traverse</td>
<td>*</td>
</tr>
<tr>
<td>Nobles</td>
<td>7.60%</td>
<td>Wabasha</td>
<td>*</td>
</tr>
<tr>
<td>Norman</td>
<td>*</td>
<td>Wadena</td>
<td>*</td>
</tr>
<tr>
<td>Olmsted</td>
<td>24.86%</td>
<td>Waseca</td>
<td>85.69%</td>
</tr>
<tr>
<td>Otter Tail</td>
<td>31.12%</td>
<td>Washington</td>
<td>9.46%</td>
</tr>
<tr>
<td>Pennington</td>
<td>34.60%</td>
<td>Watonwan</td>
<td>*</td>
</tr>
<tr>
<td>Pine</td>
<td>29.36%</td>
<td>Wilkin</td>
<td>*</td>
</tr>
<tr>
<td>Pipestone</td>
<td>63.77%</td>
<td>Winona</td>
<td>61.92%</td>
</tr>
<tr>
<td>Polk</td>
<td>68.89%</td>
<td>Wright</td>
<td>13.60%</td>
</tr>
<tr>
<td>Pope</td>
<td>49.71%</td>
<td>Yellow Medicine</td>
<td>76.56%</td>
</tr>
<tr>
<td>Ramsey</td>
<td>29.41%</td>
<td></td>
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</tr>
<tr>
<td>Red Lake</td>
<td>*</td>
<td></td>
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</tr>
<tr>
<td>Redwood</td>
<td>1.50%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Renville</td>
<td>75.42%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rice</td>
<td>81.08%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rock</td>
<td>40.00%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Roseau</td>
<td>*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>St. Louis</td>
<td>.43%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Due to insufficient sample size these counties could not be tested.*
size limitations, no test of industrial appraisals was possible.¹

Another set of Mann-Whitney tests was run with commercial appraisals eliminated from the sample. These tests also suffered from very small sample size. There were frequently more commercial appraisals than sales; thus not using appraisals eliminated the majority of commercial observations in many counties. Only 46 counties could be analyzed and of these 9 had less than 20 percent probabilities of being comparable.

To reiterate, given the small sample size and the fact that the industrial observations were almost all appraisals, it is difficult to state conclusively that a distortion exists for any particular county. Taking the broader view, however, that generally the industrial appraisals are indicative of industrial sales, the results obtained above do support the contention that commercial and industrial property should not be automatically combined.

FINDINGS:

1. Combining commercial and industrial property when the two categories do not have comparable assessment/sales ratios will have the following effects:

   a. The average assessment/sales ratio for the combined category will not be accurate for either property type.

   b. The calculated adjusted assessed values will be inaccurate. School aids will differ from the allocations which would be obtained if the property types were handled separately.

¹The Program Evaluation Division's data tape was constructed by selecting the last year of data from Revenue's 1976, 1975, and 1974 study tapes. Only the last year on each tape had sales matched with assessor's market values in the year of sale. In order for Revenue to begin processing to meet their deadlines, the last year on each Revenue tape contains only the first 10 months of sales. Thus the construction of the Program Evaluation Division's tape seriously reduced sample size. For property types with few sales, and especially for the commercial and industrial property for which certificates tend to come in very late in the year, our tape construction severely reduced the sample size. The Program Evaluation Division data tape has roughly one-third of the commercial and industrial observations contained on the Department of Revenue data tapes. The tests mentioned in the text could be more successfully attempted with a full Revenue data tape, with data matched according to our recommendation. In some areas it should be possible to test the comparability of industrial sales and appraisals.
(c) The calculated adjusted assessed value for the combined category will fluctuate over time, since the combined aggregate mean will be sensitive to the relative composition of property types in the sample. Additional sales for either property type may alter the school aid allocation.

(d) Appraisals can alter the aid allocation, regardless of their similarity to sales.

(2) Tests performed on 66 counties (21 counties could not be analyzed due to insufficient sample size) revealed that 23 counties had less than 20 percent probabilities of comparability between commercial and industrial assessment/sales ratios. These tests included appraisals in both the industrial and commercial categories.

(3) When commercial appraisals were eliminated and another series of tests was performed, only 46 counties could be analyzed. Of these, 9 counties had less than a 20 percent probability of being comparable.

RECOMMENDATIONS:

(1) Since even conservative statistical tests suggest that commercial and industrial properties cannot validly be merged for many counties, these categories should be maintained and processed separately for assessment/sales ratio and aid calculation purposes. If commercial and industrial properties are comparably assessed, sales ratios and aids will be identical whether they are combined or handled separately. If they are not comparably assessed, combining the two categories will produce assessment/sales ratios which are not accurate for either property type and will result in distorted aid allocations.

The following recommendations are contingent upon the data matching recommendation in Chapter 1.

(2) In cases where sample size is deemed insufficient for a particular property type, a fourth year of sales data should be used in preference to appraisals for expanding the sample unless Mann-Whitney tests demonstrate that appraisals have a substantially higher probability of comparability to the three-year sample than the fourth

\[ \text{No testing procedure should be used if the present data matching system is maintained. In addition, these specific testing procedures are designed to conform with our recommendations concerning calculation of adjusted assessed values. Modifications are necessary if those recommendations are not followed.} \]
Three related recommendations follow:

(a) Simultaneous use of appraisals and a fourth year of sales to enlarge the sample should be avoided.

(b) Using five years of sales data is not recommended.

(c) If Mann-Whitney tests are not feasible, a fourth year of sales data should always be used in preference to appraisals.

In cases where sample size is deemed insufficient for both commercial and industrial property, the two types may be combined if and only if the appropriate Mann-Whitney tests indicate that combining the two will yield more reliable results than using either appraisals or a fourth year of sales data to enlarge the individual samples.

**Agricultural Property**

There are currently two classifications used for agricultural property: improved agricultural land and unimproved land. Improved agricultural land is farmland with buildings present. Unimproved agricultural land refers to tracts of farmland on which there are no buildings.

Currently these two categories are combined in the study, as is presently the procedure with commercial and industrial property. To determine if combining the two may be detrimental, the Mann-Whitney test was again used. Here the sample was sufficient to allow all appraisals to be eliminated from the sample, permitting the test to be a comparison solely of improved agricultural sales vs. unimproved agricultural sales. Again, the probabilities given in

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1. Theoretical and practical considerations favor using a fourth year of sales over appraisals. The cost of obtaining and using the additional sales data is minimal since these are already on file at the Department of Revenue, while appraisals are relatively expensive to obtain. Furthermore, Mann-Whitney tests may subsequently reveal that the appraisals should not be used in the study.

2. The data on improved sales are from 1974, 1975, and 1976, with estimated market values from the year of sale matched with the sale prices. The unimproved sales are from 1976. In earlier years unimproved sales were not included in the study.
Table 6 can loosely be interpreted as the probability that the individual assessment/sales ratios for the two categories of agricultural property are comparable.

Only six counties could not be analyzed due to sample size. This group included Hennepin and Ramsey, where the number of farms sold was very small due to the urban character of the counties. Of the remaining 81 counties, 40 of these (nearly half the sample) had probabilities of being comparable of less than 20 percent.

The pattern for most counties was that the average assessment/sales ratio for unimproved agricultural land was lower than the average ratio for improved agricultural land. This suggests that assessors have not kept pace with recent high rates of inflation on farmland, causing agricultural land to be generally underassessed relative to buildings.

If this pattern is true, as our tests suggest, then two problems may occur:

(1) For a given county the combined aggregate sales ratio may vary over time, causing the adjusted assessed value and aids to vary also. For an agricultural county where land is underassessed relative to buildings, if improved agricultural sales predominate in the sample the average ratio will be high. The smaller the average size of the farms (land and buildings) that sell, the higher the average ratio will be, because building value accounts for a high proportion of total value. If in the following year unimproved agricultural sales predominate, the ratio would fall, adjusted assessed value would increase, and aids would fall.

(2) The equalization process between counties in a given year may be ineffective. Assume two identical counties with identical assessment levels, and both underassess land relative to buildings. If the first county has a majority of improved sales (land with buildings), but the second county has a majority of unimproved sales (land only), the calculated adjusted assessed values will differ.

FINDINGS:

(1) Based upon statistical tests, in nearly half the counties in the state there is evidence that improved agricultural sales and unimproved sales have different average assessment/sales ratios.

(2) Since unimproved and improved sales are currently combined in the study, when the average ratios differ the following problems can be expected:
### TABLE 6

Probabilities of Comparability for Assessment/Sales Ratio Data: Improved versus Unimproved Agricultural Sales

<table>
<thead>
<tr>
<th>County</th>
<th>Probability</th>
<th>County</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aitkin</td>
<td>21.29%</td>
<td>Fillmore</td>
<td>.19%</td>
</tr>
<tr>
<td>Anoka</td>
<td>23.96%</td>
<td>Freeborn</td>
<td>19.59%</td>
</tr>
<tr>
<td>Becker</td>
<td>.03%</td>
<td>Goodhue</td>
<td>13.46%</td>
</tr>
<tr>
<td>Beltrami</td>
<td>23.51%</td>
<td>Grant</td>
<td>94.64%</td>
</tr>
<tr>
<td>Benton</td>
<td>17.29%</td>
<td>Hennepin</td>
<td>*</td>
</tr>
<tr>
<td>Big Stone</td>
<td>4.49%</td>
<td>Houston</td>
<td>24.03%</td>
</tr>
<tr>
<td>Blue Earth</td>
<td>87.38%</td>
<td>Hubbard</td>
<td>.34%</td>
</tr>
<tr>
<td>Brown</td>
<td>35.26%</td>
<td>Isanti</td>
<td>89.28%</td>
</tr>
<tr>
<td>Carlton</td>
<td>16.13%</td>
<td>Itasca</td>
<td>53.76%</td>
</tr>
<tr>
<td>Carver</td>
<td>55.46%</td>
<td>Jackson</td>
<td>49.99%</td>
</tr>
<tr>
<td>Cass</td>
<td>23.89%</td>
<td>Kanabec</td>
<td>4.91%</td>
</tr>
<tr>
<td>Chippewa</td>
<td>19.35%</td>
<td>Kandiyohi</td>
<td>59.30%</td>
</tr>
<tr>
<td>Chisago</td>
<td>14.66%</td>
<td>Kittson</td>
<td>.00%</td>
</tr>
<tr>
<td>Clay</td>
<td>10.05%</td>
<td>Koochiching</td>
<td>9.46%</td>
</tr>
<tr>
<td>Clearwater</td>
<td>56.69%</td>
<td>LacQui Parle</td>
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</tr>
<tr>
<td>Cook</td>
<td>*</td>
<td>Lake</td>
<td>*</td>
</tr>
<tr>
<td>Cottonwood</td>
<td>*</td>
<td>Lake of the</td>
<td>3.08%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Woods</td>
<td></td>
</tr>
<tr>
<td>Crow Wing</td>
<td>19.17%</td>
<td>LeSueur</td>
<td>79.89%</td>
</tr>
<tr>
<td>Dakota</td>
<td>85.01%</td>
<td>Lincoln</td>
<td>15.42%</td>
</tr>
<tr>
<td>Dodge</td>
<td>70.57%</td>
<td>Lyon</td>
<td>21.93%</td>
</tr>
<tr>
<td>Douglas</td>
<td>80.47%</td>
<td>McLeod</td>
<td>95.02%</td>
</tr>
<tr>
<td>Faribault</td>
<td>50.22%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Due to insufficient sample size these counties could not be tested.*

42
<table>
<thead>
<tr>
<th>County</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mahnomen</td>
<td>18.06%</td>
</tr>
<tr>
<td>Marshall</td>
<td>41.80%</td>
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<tr>
<td>Martin</td>
<td>20.61%</td>
</tr>
<tr>
<td>Meeker</td>
<td>85.97%</td>
</tr>
<tr>
<td>Mille Lacs</td>
<td>.15%</td>
</tr>
<tr>
<td>Morrison</td>
<td>.25%</td>
</tr>
<tr>
<td>Mower</td>
<td>30.49%</td>
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<tr>
<td>Murray</td>
<td>48.68%</td>
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<tr>
<td>Nicollet</td>
<td>.84%</td>
</tr>
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<td>Nobles</td>
<td>.38%</td>
</tr>
<tr>
<td>Norman</td>
<td>14.83%</td>
</tr>
<tr>
<td>Olmsted</td>
<td>4.67%</td>
</tr>
<tr>
<td>Otter Tail</td>
<td>19.32%</td>
</tr>
<tr>
<td>Pennington</td>
<td>19.03%</td>
</tr>
<tr>
<td>Pine</td>
<td>39.58%</td>
</tr>
<tr>
<td>Pipestone</td>
<td>20.59%</td>
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<td>Polk</td>
<td>.00%</td>
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<tr>
<td>Pope</td>
<td>15.16%</td>
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<tr>
<td>Ramsey</td>
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<tr>
<td>Red Lake</td>
<td>47.05%</td>
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<tr>
<td>Redwood</td>
<td>21.33%</td>
</tr>
<tr>
<td>Renville</td>
<td>4.36%</td>
</tr>
<tr>
<td>Rice</td>
<td>50.49%</td>
</tr>
<tr>
<td>Rock</td>
<td>4.34%</td>
</tr>
<tr>
<td>Roseau</td>
<td>.01%</td>
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<tr>
<td>St. Louis</td>
<td>40.95%</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Scott</td>
<td>9.63%</td>
</tr>
<tr>
<td>Sherburne</td>
<td>92.63%</td>
</tr>
<tr>
<td>Sibley</td>
<td>18.84%</td>
</tr>
<tr>
<td>Stearns</td>
<td>.11%</td>
</tr>
<tr>
<td>Steele</td>
<td>33.02%</td>
</tr>
<tr>
<td>Stevens</td>
<td>51.75%</td>
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<tr>
<td>Swift</td>
<td>22.88%</td>
</tr>
<tr>
<td>Todd</td>
<td>4.60%</td>
</tr>
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<td>Traverse</td>
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<tr>
<td>Wabasha</td>
<td>7.78%</td>
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<td>Wadena</td>
<td>22.50%</td>
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<tr>
<td>Waseca</td>
<td>14.48%</td>
</tr>
<tr>
<td>Washington</td>
<td>70.55%</td>
</tr>
<tr>
<td>Watonwan</td>
<td>78.33%</td>
</tr>
<tr>
<td>Wilkin</td>
<td>43.77%</td>
</tr>
<tr>
<td>Winona</td>
<td>9.92%</td>
</tr>
<tr>
<td>Wright</td>
<td>13.91%</td>
</tr>
<tr>
<td>Yellow Medicine</td>
<td>61.58%</td>
</tr>
</tbody>
</table>

* Due to insufficient sample size these counties could not be tested.
(a) The assessment/sales ratios and aids may fluctuate over time.

(b) In a given year, the equalization process between counties may be ineffective.

(3) The differences between the assessment/sales ratios for the two categories appear to be due to differences in assessment levels between land and buildings.

The problems presently encountered with agricultural properties in the assessment/sales ratio studies have no simple solutions: Until the feasibility and accuracy of alternative procedures can be determined, there is no better alternative than to continue present procedures. A further discussion of the problems and several alternatives to the present procedures appear in Appendix D. Unfortunately, determining the accuracy and feasibility of alternatives will be difficult.

SELECTING AGGREGATE ASSESSMENT/SALES RATIOS

The general issue of selecting aggregate assessment/sales ratios for use in calculating adjusted assessed values is of considerable importance. Particularly in cases where the number of sales in a given property category is small, the aggregate assessment/sales ratio of the city/town sample may be an inadequate indicator of the true ratio for the city or town. In this section present Department guidelines for selecting an aggregate ratio for use in the eventual calculation of adjusted assessed values are outlined, the possibility of errors in computer programs implementing these guidelines is explored, and the adequacy of present guidelines is examined.

PRESENT GUIDELINES

When matching limited market values with aggregate assessment/sales ratios on the final tape verification listing, the following general guidelines are used for apartment, seasonal recreational, and commercial and industrial property types:

(1) If there are one or two sales (or appraisals) for a given property type in a town or city, but these sales represent more than 15 percent of the total assessed value for that property type, the city or town aggregate assessment/sales ratio for that property type is used.

(2) If there are two sales (or appraisals) but these sales comprise less than 15 percent of the total assessed value
for that property type, the county-wide aggregate assessment/sales ratio for that property type is used.

(3) If there are no sales (or appraisals), or one sale which represents less than 15 percent of the total assessed value for that property type, the ratio is manually assigned. Generally, either the county-wide ratio or a ratio from another property type is used.

(4) If there are three or more sales in a property category, the city or town ratio is used.

For agricultural property, only a county-wide ratio is generated. For residential property, if there are less than three sales, a county-wide ratio is selected. If there are three or more sales, the city or town residential ratio is used.

An examination of the final tape verification listing for a sample of several school districts in the 1976 study revealed departures from the above guidelines which may be the result of flaws in the computer programs:

(1) For one township in a particular school district, a computer program selected the wrong assessment/sales ratio for use in calculating the indicated market value or residential property. In this case there were 113 sales, yet the county-wide ratio was substituted for the township ratio. An examination of this township in the previous year's study revealed the same occurrence.

(2) In another town, the total limited market value of apartment properties was never selected from the data tape and never used.

If our recommended changes in the calculation of adjusted assessed values are adopted, the programs currently used to match limited market values with aggregate assessment/sales ratios will be unnecessary. Instead, average total assessed values would be matched with aggregate assessment/sales ratios. If the changes are not made, then the present computer programs used for school aid purposes should be carefully examined and corrected.
INADEQUACY OF PRESENT GUIDELINES

A test of the adequacy of the present guidelines concluded that three sales (or appraisals) do not seem to constitute a sufficient sample to justify using the city/town ratio. The following is a brief description of the procedure used in this test.

First, all towns or cities which had three commercial sales were selected from the Program Evaluation Division's data tape. There were twenty towns in this sample. Next the city or town aggregate assessment/sales ratio was calculated based on the three sales, and a confidence interval was developed. A confidence interval may be interpreted as a range of values with a given probability of containing the true population mean. For instance, for a given property type in a town, the probability might be 95 percent that the true value lies between 50 percent and 80 percent. This range represents the "95 percent confidence interval." Using the data from each town in the sample, both 95 percent and 80 percent confidence intervals were calculated. The 95 percent intervals are always broader than the 80 percent intervals because the range must be increased to increase the probability that the true value actually falls within the interval.

Given the variability of the individual assessment/sales ratios, and the very small sample size (only three sales), the confidence intervals were very wide. For 95 percent confidence intervals a spread of fifty percentage points was not uncommon, and some towns had far larger intervals. One town had a range of 6 percent to 159 percent; another town had a range of -9 percent to 126 percent. For 80 percent confidence intervals the typical spread was 20 to 30 points, with some towns again greatly exceeding this range. In the two towns mentioned previously the 80 percent confidence intervals were 49 to 116 percent, and 29 to 88 percent respectively.

1 The procedure to be described in the text is essentially a byproduct of the Program Evaluation Division's development of procedures for calculating aggregate assessment/sales ratio confidence intervals. A more refined approach would permit the handling of the county-wide mean as an estimate rather than as a parameter, and would also incorporate the variability of this estimate. In addition, the correlation between the two aggregate ratio and the county-wide ratio must be considered. The two are correlated because the town data is a subset of the data used to calculate the county-wide average. A preferred test would be a generalized t test which allows separate variance estimates and a correlation (covariance) term.

2 Due to possible problems with the use of appraisals, only sales were used.

3 The size of any confidence interval of a specified probability will depend upon the sample size and the variability of the individual assessment/sales ratios in the sample.
Since the confidence intervals are very broad, little faith can be placed in these city or town ratios. In the first case mentioned above the aggregate assessment/sales ratio calculated from the sample was 82.5 percent; however, the strongest statement one can safely make is that the actual community ratio for commercial property probably lies somewhere between 6 and 159 percent. Therefore 82.5 percent could be a very poor estimate of the true community ratio. Another problem is that ratios generated from very small samples could vary considerably over time.

The test of whether using city/town ratios instead of county aggregate ratios is reasonable, when there are only three sales in the city/town sample, is whether the city/town confidence intervals contain the county aggregate ratios. If the county ratios are generally found to lie within the confidence intervals, then there is little assurance that the true city/town ratios differ from the county ratios. Thus, using the county ratios is at least justifiable. Further, in light of the relative stability of county ratios and considering the weaknesses of city/town ratios, use of the county ratios may be preferable.

After calculating 95 and 80 percent confidence intervals for each of the twenty cities and towns, the county-wide aggregate assessment/sales ratios for commercial property were calculated. Each city/town confidence interval and the corresponding county aggregate assessment/sales ratio were then examined to determine if the interval contained the county-wide ratio.

For the sample of twenty cities or towns where three sales occurred, nineteen times out of twenty the 95 percent confidence interval contained the county-wide commercial aggregate ratio. For 80 percent confidence intervals, fourteen times out of twenty the county-wide average was contained in the interval.

Another sample was drawn, this time selecting cities or towns with five sales. This time both the 95 percent and 80 percent confidence intervals contained the county-wide aggregate mean in 80 percent of the cases. When cities or towns with six sales were selected, the 95 percent confidence interval contained the county-wide average in 67 percent of the cases. The 80 percent confidence intervals contained the county-wide average in 50 percent of the cases.

While more sophisticated tests are possible, the findings derived from the above procedure strongly indicate the county-wide average should be used when there are only three sales or appraisals in a town sample. Until further research is performed to permit a more precise guideline, it appears that a cutoff of six sales or appraisals is preferable to the present guideline.

**FINDINGS:**

(1) In the 1976 studies, for one township in a particular
school district, one of the computer programs selected the wrong assessment/sales ratio for use in equalizing the residential property. Whenever there are more than 3 sales for a given property type in a town, the town ratio is normally selected. In this case there were 113 sales, yet the county-wide ratio was substituted for the town ratio. An examination of this township in the previous year's study revealed the same occurrence.

(2) In another town, the total limited market value of apartment properties was never selected from the data tape and never used.

(3) Three sales (or appraisals) do not appear to constitute a sufficient sample to warrant use of the city/town ratio. A sample of towns with three commercial sales typically had 95 percent confidence intervals with a range of fifty points. The interval for one town was over 150 points, from 6 to 159 percent. The typical range for 80 percent confidence intervals was 20 to 30 points. In nineteen out of twenty cities or towns the 95 percent confidence interval contained the county-wide aggregate ratio. For 80 percent confidence intervals, fourteen times out of twenty the county-wide average was contained in the interval. Thus with only three sales it can not be generally assumed that the town aggregate ratio differs from the county-wide ratio. Furthermore, ratios based upon three sales or appraisals will be variable over time and may be poor indicators of the true city or town ratio.

RECOMMENDATIONS:

(1) If our recommendation concerning the procedure for calculating adjusted assessed values is adopted, the present computer programs used to match limited market values with assessment/sales ratios will be unnecessary. If the present system for calculating adjusted assessed values is retained, these computer programs should be carefully examined and corrected if programming errors exist.

(2) General use of the city or town aggregate assessment/sales ratio when the sample consists of only three sales or appraisals should be discontinued. If the Department continues to use a guideline based upon the number of observations in the sample, a rule that there be at least six sales or appraisals seems more appropriate. Further research in this area is warranted.

(3) For certain property types, the Department of Revenue should investigate the feasibility of incorporating a statistical test into their computer programs which will automatically determine whether a given city or town ratio for
the given property type is significantly different from the county-wide aggregate ratio. If the city or town ratio for the given property type is significantly different, the city or town ratio can automatically be selected. If not, the county-wide ratio can be used.
CHAPTER V
CONCLUSION

This conclusion briefly mentions the difficulty in estimating the impact of present procedures upon aid allocations or the net changes that will occur if our recommendations are adopted. This is followed by a short discussion of procedures which can be used if the assessment/sales ratio study is tailored to indicate assessor performance, rather than for aid distribution. Finally, we offer a few suggestions for further research.

MEASURING THE IMPACT OF PROGRAM EVALUATION DIVISION RECOMMENDATIONS

This report of the procedures used in the Department of Revenue assessment/sales ratio studies has addressed many problems, from data screening to difficulties in calculating adjusted assessed values. Given the number of problems and their accompanying biases, it is impossible for the Program Evaluation Division to estimate the potential net impact of our recommendations on any district or property type. This can only be estimated by performing a parallel study incorporating our recommendations.

TAILORING THE STUDIES FOR DIFFERENT PURPOSES

While tailoring an assessment/sales ratio study to one form of use must involve sacrificing optimal effectiveness in other uses, care has been taken to ensure that the recommendations are basically compatible with the various uses of the Department's studies. The primary emphasis of our recommendations is to provide an acceptable basis for school aid distribution. If the primary emphasis is to develop indicators of assessor consistency and uniformity, for use by local assessors or by the Department, the following should be considered:

(1) Only assessment/sales ratios based upon estimated market values need to be calculated. There is no need to screen or use limited market value data.

(2) There is no need to use a multi-year sample if data from one year, or even a fraction of one year, are sufficient.

(3) There is a need to test for changes in assessor performance. It is obviously inappropriate to use a multi-
year average of assessment/sales ratios if this does not reflect recent performance.

The second suggestion above implies that it is not necessary to use all potential data if a small subsample is sufficient. Besides the simple tests which appear in Chapter IV dealing with confidence intervals, additional samples of residential property from various cities and towns were examined, and confidence intervals were developed. These samples ranged in size from several sales to a few thousand. Based on the confidence intervals, it definitely appears that there is very little to be gained by screening and using more than 500 sales for a given town and property type. If the purpose of a study is to examine assessor performance and several hundred sales are available for a property type and town in a year, a subsample of the data could be used. A similar procedure is possible in studies used for school aid purposes, except that a comparable number of years for all districts and property types should be used, if possible. In this case a subsample from several year's data can be drawn.  

FURTHER RESEARCH

The Program Evaluation Division's study has not examined nor reached conclusions concerning all important aspects of the assessment/sales studies. In Appendix D, alternative procedures for the agricultural sample are suggested. Additional research is necessary to determine the feasibility of these methods. The effect of the recommendations in this report upon local government aids also requires study. A further issue which was not addressed here is the optimal number of years in the data base. A multi-year sample is definitely required, considering the sparse number of sales for many property types in out-state areas. However, the exact number of years to be used requires further investigation. Using two years instead of the current three years may make estimates of adjusted assessed values generally more indicative of current taxable property wealth, but greater reliance would be placed on county-wide ratios, and the reduced sample size might

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1 In studies used for school aid determination, our recommendation for calculating adjusted assessed values is designed to compensate for changes in assessor performance, making such testing generally unnecessary.

2 Given our suggested adjusted assessed value calculation, the subsample should maintain the basic pattern of sales as they occur in the sample. If 50 percent of the sales occurred in the first year and 25 percent in each of the most recent years, 50 percent of the subsample should be drawn from the first year, and 25 percent from each of the most recent years.
increase variability. We did not address this issue; our recommendations are consistent with any multi-year study. Substantial work remains to be done in the area of test procedures, especially in developing tests of assessor uniformity. Research on the economic impacts of reassessment and classification ratio changes is needed. Finally, the effects of different types of financing on property sale prices is an important subject for study.
APPENDIX A
DATA MATCHING METHODOLOGY

The Department currently bases its aggregate ratios on a three year base of sales and appraisals, which are compared to the most recent year's assessor's market values. This results in the following problems:

1. As assessor's values increase during the three year period due to inflation, the individual ratios calculated for the earlier years are systematically overstated. This causes the three year aggregate mean to become overstated as well. See Section 1.

2. In districts with rapidly increasing assessments, the three year average ratio is overstated even more dramatically than in districts with more gradual increases in assessment. Districts with comparable assessment practices but experiencing different rates of inflation will have different aggregate means and thus will be treated differently for state aid purposes. See Section 2.

3. Under current data matching, if there is inflation and assessor performance is unchanging, an abnormally high number of sales in a given year will cause a "ripple effect" in the aggregate mean over time. As these sales enter the sample the mean will fall, and then it will increase until these sales leave the three year sample, at which time the mean will again fall. Since the actual assessment level is unchanged, while the calculated aggregate mean changes over time, the aggregate mean will not be a proper indicator of assessment level. See Section 3.

4. The current averages are unacceptable measures. They are not, in general, true measures of central tendency, since it is possible for the calculated means to be higher than any true assessment/sales ratio in the population. This results from matching old sales with new assessments, a procedure which invalidates much of the data. See Section 4.

5. All currently published measures of dispersion are invalid indicators of assessment uniformity. The present data matching produces a mixture of actual variability and methodological biases. Even if there is no actual variability, given inflation the Department's procedure will produce coefficients of dispersion which increase with the rate of inflation. In addition, the coefficient of dispersion is influenced by when sales enter the sample. See Section 5.
6. The Department's use of the aggregate mean is intended to weigh more expensive property more heavily in the calculation of the final sales ratio. However, any sale (including high priced sales) which are relatively under-assessed may in later years raise the aggregate mean, while logic dictates that such low ratios should bring the aggregate mean down. A high priced sale, if under-assessed, will lower the aggregate mean when it is included in the first year of the three year sample. In subsequent years, as this sale is combined with newer assessments, the aggregate mean will increase. Then as this high priced sale leaves the sample after 3 years, the mean will fall. See Section 6.

7. The current methodology imputes changes in assessment levels when in fact none exist. This fault will invalidate the results of most statistical tests performed on such data. In fact, the use of statistical tests could lead to wide fluctuations in calculated average assessment/sales ratios. See Section 7.
SECTION 1
INFLATION BIASES MEANS UPWARD

In Chapter I several serious problems caused by the present system of data matching are discussed. The material in this appendix illustrates these issues. In Table 7 reasonable sale prices are assumed and there is 15 percent inflation of which the assessor is cognizant. (Other inflation rates are considered later.) Due to inflation, average sale prices in 1976 are higher than in 1975, and 1977 prices are higher than 1976 prices. This table is very similar to Exhibit 1 on page 11 of the text; the only difference is that Table 7 incorporates a large number of sales. The assessor has been assessing with perfect uniformity at 80 percent of current market value, and to maintain this level he/she increases estimated market values by 15 percent per year.\(^1\) Therefore, regardless of what property sells, or when it sells, the individual ratios are all 80 percent, and any measure of dispersion will be zero. However, if the means and measures of dispersion on these samples are calculated according to the Department's methodology, the means will not be 80 percent and there will appear to be dispersion. These differences are due solely to methodological flaws.

The critical error is Revenue's matching of old sales with the most recent assessor's market values. The result of this matching is depicted in Table 8. The first 1975 sale was $20,000; this property had an assessor's market value of $16,000 when sold. One year later the assessor would have increased this figure by 15 percent, to $18,400 ($16,000 \times 1.15 = $18,400). In 1977, the assessor's value on this property is $16,000 \times 1.15 \times 1.15 = $21,160. When the Department places this 1975 sale in their study, it is handled as though it were assessed at 105.8 percent ($21,160/ $20,000 = 105.8\%). The 1976 sale properties would be matched with assessor's market values which are 15 percent higher than when the property sold; the study would handle them as though they were assessed at 92 percent of sale price. Only the sales in the last year, 1977, are matched with assessor's market values for the year of sale, thus they are the only sales included with an 80 percent ratio. The computer would calculate averages based on individual assessment/sales ratios ranging from 80 percent to 105.8 percent. The aggregate mean is 91 percent, the arithmetic mean is 92.6 percent, and the median is 92 percent, yet all property has assessor's values which are 80 percent of current market value. Furthermore, while there is actually no dispersion, the Department of Revenue's methodology would result in a coefficient of dispersion for this three year sample of 9.35 percent.

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\(^1\)This example relates to estimate market values, not limited values.
TABLE 7
HYPOTHETICAL 1977 SALES RATIO STUDY
DATA GROUPED AS PER PROGRAM EVALUATION DIVISION METHODOLOGY

<table>
<thead>
<tr>
<th>1975 SAMPLE</th>
<th>1976 SAMPLE</th>
<th>1977 SAMPLE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assessor's</td>
<td>Assessor's</td>
<td>Assessor's</td>
</tr>
<tr>
<td>Sale Price</td>
<td>Market Value</td>
<td>Market Value</td>
</tr>
<tr>
<td>1975</td>
<td>1975</td>
<td>1975</td>
</tr>
<tr>
<td>$20,000</td>
<td>$16,000</td>
<td></td>
</tr>
<tr>
<td>28,000</td>
<td>22,400</td>
<td></td>
</tr>
<tr>
<td>40,000</td>
<td>32,000</td>
<td></td>
</tr>
<tr>
<td>80,000</td>
<td>64,000</td>
<td></td>
</tr>
<tr>
<td>37,000</td>
<td>29,600</td>
<td></td>
</tr>
<tr>
<td>42,000</td>
<td>33,600</td>
<td></td>
</tr>
<tr>
<td>39,000</td>
<td>31,200</td>
<td></td>
</tr>
<tr>
<td>50,000</td>
<td>40,000</td>
<td></td>
</tr>
</tbody>
</table>

| Assessor's  | Assessor's  | Assessor's  |
| Sale Price  | Market Value | Market Value |
| 1976        | 1976        | 1977        |
| $100,000    | $80,000     | $120,000    |
| 80,000      | 64,000      | 100,000     |
| 50,000      | 40,000      | 42,000      |
| 62,000      | 49,600      | 35,000      |
| 32,000      | 25,600      | 43,000      |
| 25,000      | 20,000      | 51,000      |
| 42,000      | 33,600      | 32,000      |
| 35,000      | 28,000      | 60,000      |

| Assessor's  | Assessor's  | Assessor's  |
| Sale Price  | Market Value | Market Value |
| 1977        | 1977        | 1977        |
| $12,000     | $96,000     | $120,000    |
| 100,000     | 80,000      | 100,000     |
| 42,000      | 33,600      | 42,000      |
| 35,000      | 28,000      | 35,000      |
| 43,000      | 34,400      | 43,000      |
| 51,000      | 40,800      | 51,000      |
| 32,000      | 25,600      | 32,000      |
| 60,000      | 48,000      | 60,000      |

Aggregate mean: 80%
Arithmetic mean: 80%
Median: 80%
Coefficient of dispersion: 00%

1 The aggregated mean is the sum of the assessor's market values divided by the sum of the sale prices.
To calculate the arithmetic mean, first divide the assessor's market value by the sale price of each unit of property. Add these ratios, then divide by the number of sales in the three year sample.
To calculate the median, divide each assessor's market value by its sale price. Arrange these ratios from smallest to largest and select the middle ratio in the series.
<table>
<thead>
<tr>
<th>1975 Sample</th>
<th>1976 Sample</th>
<th>1977 Sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sale Price</td>
<td>A.M.V. 1</td>
<td>A.M.V. 3</td>
</tr>
<tr>
<td>$20,000</td>
<td>$16,000</td>
<td>$21,160</td>
</tr>
<tr>
<td>$28,000</td>
<td>$22,400</td>
<td>$29,624</td>
</tr>
<tr>
<td>$40,000</td>
<td>$32,000</td>
<td>$42,320</td>
</tr>
<tr>
<td>$62,000</td>
<td>$49,600</td>
<td>$57,040</td>
</tr>
<tr>
<td>$32,000</td>
<td>$25,600</td>
<td>$29,440</td>
</tr>
<tr>
<td>$25,000</td>
<td>$20,000</td>
<td>$23,000</td>
</tr>
<tr>
<td>$42,000</td>
<td>$33,600</td>
<td>$38,640</td>
</tr>
<tr>
<td>$39,000</td>
<td>$31,200</td>
<td>$41,262</td>
</tr>
</tbody>
</table>

1 A.M.V. = Assessor's Market Value
2 This first unit of property had an assessor's market value of $16,000 in 1975. In 1977, this inflated to $21,160 ($16,000 x 1.15 x 1.15 = $21,160).
3 Data grouped as per Program Evaluation Division methodology.
4 Data grouped as per Department of Revenue methodology.

Measures of central tendency and dispersion calculated as per Department of Revenue methodology:

- Aggregate mean: 91%
- Median: 92%
- Arithmetic mean: 92.6%
- Coefficient of dispersion: 9.35%
SECTION 2

BIAS DIFFERS WITH THE INFLATION RATE

An implication of the previous analysis is that districts which actually have identical assessment levels and dispersion may not have identical statistics under the current system. The Department would calculate different means and measures of dispersion if inflation rates varied between districts. In Table 9 the data in Table 7 has been used to generate means and coefficients of dispersion as per the Department of Revenue's methodology under three inflation rates: 5 percent, 10 percent, and 15 percent. The actual level of assessment in each case is 80 percent, and sale prices, sample size, and the distribution of sales within the sample are the same as in Table 7 and Table 8. As expected, the higher the inflation rate, the higher the means. If the rate of inflation is 15 percent, 1975 sales enter the sample as though they were assessed at 105.8 percent, 1976 sales at 92 percent, and 1977 sales at 80 percent. With 10 percent inflation, 1975 sales enter at 96.8 percent, 1976 sales at 88 percent, and 1977 sales at 80 percent. Thus two districts with identical assessment levels and dispersion may not have similar statistics.

The current statistics will lead to state aid misallocations because the means in each district are not accurate, nor are they biased by equal proportions. Furthermore, using the current statistics as a basis for Commissioner's orders for district-wide reassessments is highly questionable, since real differences are indistinguishable from methodological biases, except in the most flagrant cases.
<table>
<thead>
<tr>
<th>Inflation Rate</th>
<th>5%</th>
<th>10%</th>
<th>15%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggregate Mean</td>
<td>83.6%</td>
<td>87.2%</td>
<td>91%</td>
</tr>
<tr>
<td>Arithmetic Mean</td>
<td>84.1%</td>
<td>88.3%</td>
<td>92.6%</td>
</tr>
<tr>
<td>Median</td>
<td>84%</td>
<td>88%</td>
<td>92%</td>
</tr>
<tr>
<td>Coefficient of Dispersion</td>
<td>3.25%</td>
<td>6.36%</td>
<td>9.35%</td>
</tr>
</tbody>
</table>

\(^1\) All property had an individual assessment/sales ratio of 80 percent when sold. The results are based on the original sales in Table 7.
SECTION 3
"RIPPLE EFFECT" DUE TO SAMPLE DISTRIBUTION

The interaction of the distribution of sales within the sample and inflation, even a steady rate of inflation, will cause a "ripple effect" in a district's aggregate mean over time. This is illustrated in Table 10. The first column gives the results obtained from the data in Table 7 if the rate of inflation is 10 percent. Suppose that in 1977 there is an unusually high number of sales, illustrated here by doubling the number of sales in that year. Since this is the most recent year of the study, these sales will enter the sample with 80 percent individual ratios, pulling the aggregate mean ratio down to 85.2 percent. If instead 1976 sales are doubled to simulate the effect of a large middle year in the sample, the aggregate mean increases since these sales enter with 88 percent ratios. Doubling the 1975 sales, the mean increases even further since these sales enter at 96.8 percent, pulling the aggregate mean up to 89.2 percent. Finally, when the year with the abnormally high number of sales is dropped from the study, the means would drop toward their original level.

The effects illustrated in Table 10 are similar to those which would be observed if 1977 had an abnormally large number of sales and its influence was observed through the 1977, 1978, and 1979 studies.
**TABLE 10**

EFFECT OF THE DISTRIBUTION OF SALES IN THE SAMPLE GIVEN ASSESSED VALUES INFLATE AT 10% PER YEAR, DATA GROUPED AS PER DEPARTMENT OF REVENUE METHODOLOGY

<table>
<thead>
<tr>
<th></th>
<th>Original Sample 1977 Sales</th>
<th>Doubling 1977 Sales</th>
<th>Doubling 1976 Sales</th>
<th>Doubling 1975 Sales</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggregate Mean</td>
<td>87.2%</td>
<td>85.2%</td>
<td>88.2%</td>
<td>89.2%</td>
</tr>
<tr>
<td>Arithmetic Mean</td>
<td>88.3%</td>
<td>86.2%</td>
<td>88.2%</td>
<td>90.4%</td>
</tr>
<tr>
<td>Median</td>
<td>88%</td>
<td>84%</td>
<td>88%</td>
<td>92.4%</td>
</tr>
<tr>
<td>Coefficient of Dispersion</td>
<td>6.36%</td>
<td>7.38%</td>
<td>4.77%</td>
<td>6.93%</td>
</tr>
</tbody>
</table>

1. These results are from Table 9.

2. To examine the effect of sample distribution, we first maintain the same sale prices as in previous tables for 1975 and 1976 and double 1977 sales by assuming that instead of one $120,000 sale there are two $120,000 sales, two $100,000 sales, two $42,000 sales, etc. This way the results are not influenced by the addition of sales of different numerical values than existed in the original sample. In the third column, where the number of 1976 sales are doubled, the original eight 1975 sales and the original 1977 sales are used. For the fourth column the number of 1975 sales is doubled and the original eight 1976 sales and 1977 sales are used.

3. In situations where there are an even number of sales, the median was approximated by averaging the middle values.
SECTION 4
CURRENT DATA MATCHING DESTROYS THE VALIDITY OF THE AVERAGES

To begin to bring some validity to the assessment/sales ratio studies, the sales in each year must be matched with assessments in that year, as depicted in Table 7. Only this approach would yield an accurate reflection of assessment performance. The current statistics of the Department of Revenue exhibit a very basic logical flaw. Any average, whether it is mean, aggregate mean, or median is by definition a measure of central tendency. A mean cannot lie outside the range of values in the sample. Yet this is exactly the type of result obtained in Table 8. All property was assessed at 80 percent of current market value, but the Department's methodology consistently results in measures of central tendency which are greater than 90 percent.

These results occur because the averages were based on some ratios of 105.8, some of 92, and some of 80 percent. However, the 105.8 percent and 92 percent ratios are not valid data. For example, the 105.8 percent ratios are caused by matching 1975 sales with 1977 assessor's market values. However, the 1977 assessment is the assessor's estimate of the market value of this property in 1977; it is not a second chance at estimating its 1975 value. Furthermore, whenever assessor's market values are matched with prior sales, the assessor knows the sale price which will be compared to his values, before the new assessor's market values are set. Therefore such ratios are not indicators of the assessor's ability, and are extremely questionable indicators of the community's assessment level.
SECTION 5
CURRENT DATA MATCHING DESTROYS THE VALIDITY OF DISPERSION MEASURES

In addition to its effect on the measures of central tendency, the current method of data matching destroys the validity of calculated measures of dispersion. In Table 9, the coefficient of dispersion has been calculated as per current methodology assuming three different inflation rates. The true dispersion is zero, but as currently calculated the measure tends to increase with inflation. With 5 percent inflation the individual ratios on the oldest sales are 88.2 percent, on the most recent sales they are 80 percent. With 15 percent inflation the ratios range from 105.8 percent to 80 percent, resulting in larger calculated coefficients of dispersion as inflation increases. With 5 percent inflation the coefficient of dispersion is 3.25 percent, while with 15 percent inflation this index is 9.35 percent.

The distribution of sales by year within the sample also affects the coefficients of dispersion. Table 10 depicts the effect of a constant 10 percent inflation rate as the distribution of sales within the sample changes. Again, there is actually no dispersion, yet the calculated coefficient varies depending upon which year has the abnormal number of sales. If 1977 sales are doubled, the number of sales coming in with 80 percent individual ratios is increased. Since the erroneous median and arithmetic mean differ from 80, being 84 and 86.2 percent respectively, the apparent dispersion is high (7.38%). If 1976 sales are doubled instead, these sales enter the study with 88 percent ratios, which equals the median; thus the coefficient of dispersion falls to 4.77 percent. If 1975 sales are doubled, they enter at 96.8 percent, differing substantially from the erroneous 88 percent median, increasing the coefficient of dispersion to 6.93 percent.

In any actual study, all dispersion measures currently published are a contaminated mixture of actual assessment variability and methodological biases. Since the sample sizes for each year will vary both within and between districts (through no fault of the assessors), these methodological biases will have different effects on each district. A high dispersion statistic under the Department's methodology may in reality indicate nothing more than a unique sales distribution by year, while a low dispersion statistic in another district may mask bona fide dispersion in assessment levels. Therefore, given the current data matching methodology, these statistics cannot be used as indicators of assessment uniformity.
SECTION 6

THE AGGREGATE MEAN, GIVEN CURRENT METHODOLOGY

The aggregate mean is a sales-price-weighted average. In other words, assessment/sales ratios for high priced properties will have a stronger weight in the determination of the community-wide aggregate mean. Logically, property with a high individual assessment/sales ratio should pull the aggregate mean up, and a low ratio on such property should pull the aggregate mean down.

Due to current methodology the aggregate means as calculated do not necessarily have these characteristics. The effect of a sale may depend upon when the sale enters the study. If the sale is an old one, a high priced property with low assessment can increase the aggregate mean. Using the information in Table 8, where a 15 percent inflation rate was assumed, suppose that one more sale had occurred in 1975, e.g., a $150,000 property with 1975 assessor's value of $117,000, or 78 percent instead of 80 percent. If the assessor applies the same 15 percent inflation adjustment to this property, then in 1977 it will be assessed at $154,732.50. In the 1977 study, when this 1975 sale price is combined with the 1977 assessor's market value, the effective ratio for this property is not 78 percent ($117,000 \div $150,000 = 78\%) but 103 percent ($154,732.50 \div $150,000 = 103\%). Therefore, if we include this additional sale and recalculate the aggregate mean, it would increase from 91 percent as given in Table 8, to 92.4 percent. This occurs despite the fact that this additional property is underassessed. On the other hand, if this $150,000 sale had occurred in 1977, its individual ratio would be 78 percent, thus reducing the aggregate mean from 91 percent to 89.7 percent.

Therefore, in current studies underassessed properties may first lower the aggregate mean, and then raise the aggregate mean in subsequent studies, even if these properties remain underassessed. The aggregate mean is therefore made less stable over time than a properly calculated measure, and it cannot be considered a meaningful sales-weighted average.
SECTION 7
EFFECT OF PRESENT DATA MATCHING
UPON STATISTICAL TESTING

The current methodology imputes changes in assessment levels, where in fact none exist. In Table 7 all properties are actually assessed at 80 percent of current market value, but due to inflation the Department's methodology implicitly assumes that in 1975 the assessor was assessing at 105.8 percent, at 92 percent in 1976, and at 80 percent in 1977. This example serves to illustrate a comment concerning statistical tests mentioned in Chapter IV. If the system of data matching currently in use is maintained, parametric or non-parametric tests should not be used to determine whether a change in assessment performance has occurred. As the above case demonstrates, even with consistent assessment performance, as is actually the case with the data in Table 8, a Mann-Whitney test would conclude that definite changes in performance have occurred. It is also possible to identify cases where assessment performance has changed, but the Department's data matching obscures these changes. Thus given this system of data matching the results of any tests have no validity.

Furthermore, the Mann-Whitney test should never be used to determine the admissibility of a set of appraisals, given the present data matching system. If we use the example referred to in the first paragraph, if there is a group of appraisals with individual ratios clustered near 92 percent, it is very likely that a test would suggest they were permissible to add to the study. The next year these appraisals would appear to be assessed at 105.8 percent and would significantly bias the calculated measures of central tendency. In the following year these appraisals would appear to be assessed at 121.7 percent (105.8% X 1.15 = 121.7%), leading to an even worse estimate of central tendency. Finally the appraisals would be dropped from the sample and the calculated averages would plummet.

Therefore, given the present system of data matching, the use of statistical tests to determine the admissibility of a set of appraisals is inappropriate and may lead to increased fluctuations in the calculated averages.

1 The Mann-Whitney test is discussed on pp. 30 and 33.
APPENDIX B
FORMULA FOR CALCULATING ADJUSTED ASSESSED VALUES

If a single year's total assessed value is divided by a multi-year average assessment/sales ratio, and the recommended change in data matching is implemented, a district which improves its assessment level would be initially penalized. The adjusted assessed value would be overstated, and aids would fall. Over the course of a few years the adjusted assessed value would slowly regain its proper level. This problem can be corrected by use of a multi-year average total assessed value.

USING A SINGLE YEAR'S TOTAL ASSESSED VALUE

Assuming a single community and a single property type for illustration, if only the 1977 total assessed value is used the formula for the 1977 adjusted assessed value with proper data matching would be:

\[
1977 \text{ adjusted assessed value} = \frac{1977 \text{ total assessed value}}{1975 \text{ L.M.V.} + 1976 \text{ L.M.V.} + 1977 \text{ L.M.V.}} \times \frac{1975 \text{ sales} + 1976 \text{ sales} + 1977 \text{ sales}}{(\text{L.M.V.})} = \text{limited market values}
\]

To develop a numerical example, suppose the total assessed value in 1977 is $1,000,000, and the assessment/sales ratio is 50% in each year. A three year sample of sales is used to establish this assessment/sales ratio; the sum of sale prices in 1975 is $10,000, in 1976 it is $20,000, and in 1977 it is $24,000. The assessor claimed the properties which sold in 1975 were worth in total $5,000, the 1976 sales $10,000, and the 1977 sales $12,000. Following current procedure, the adjusted assessed value would be calculated by dividing the total assessed value in 1977 by the assessment/sales ratio:

\[
\left( \frac{1,000,000}{5,000 + 10,000 + 12,000} \right) = \frac{1,000,000}{50\%} = \frac{2,000,000}{50}\%
\]

66
A problem with the above formula is that if an assessor
increases the assessment level, the district will be penalized.
Suppose that in 1977 the assessor in the above case claimed the
properties that sold were worth a total of $18,000. (Instead of
assessing at 50% = $12,000 ÷ $24,000, he begins to assess at 75% =
$18,000 ÷ $24,000.) Since the assessor is now assessing at 75
percent, the total assessed value in the district will increase con­
siderably. If the total assessed value was $1,000,000 when the
average assessment level was 50 percent, it will be $1,500,000 when
the assessment level is 75 percent. (If property was assessed at
100 percent there would be $2,000,000 in assessed value. If the
assessment level is 75 percent, then the assessed value is
$2,000,000 x 75% = $1,500,000). Calculating the adjusted assessed
value, a higher total is obtained:

\[
\begin{align*}
(2) \quad \frac{1,500,000}{5,000 + 10,000 + 18,000} & = \frac{1,500,000}{61\%} = 2,459,016 \\
& = \frac{10,000 + 20,000 + 24,000}{5,000 + 10,000 + 18,000}
\end{align*}
\]

Comparing (1) and (2), due to improved assessment the district
appears to be wealthier. Therefore, the district would have to
raise more taxes locally and would receive less school aid.

THE RECOMMENDED PROCEDURE: USING A SALES-WEIGHTED
AVERAGE OF TOTAL ASSESSED VALUES

To correct this problem a sales-weighted average of total
assessed values should be used in the numerator. The recom­
mended formula is:

\[
1977 \text{ adjusted assessed value} =
\frac{(1975 \text{T.A.V.} \times S) + (1976 \text{T.A.V.} \times S) + (1977 \text{T.A.V.} \times S)}{1975 \text{L.M.V.} + 1976 \text{L.M.V.} + 1977 \text{L.M.V.}}
\]

\[
= \frac{1975 \text{sales} + 1976 \text{sales} + 1977 \text{sales}}{1975 \text{sales} + 1976 \text{sales} + 1977 \text{sales}}
\]

where \( \text{T.A.V.} = \text{total assessed value} \)
L.M.V. = limited market values

\[
S_1 = \frac{1975 \text{ sales}}{1975 \text{ sales} + 1976 \text{ sales} + 1977 \text{ sales}}
\]

\[
S_2 = \frac{1976 \text{ sales}}{1975 \text{ sales} + 1976 \text{ sales} + 1977 \text{ sales}}
\]

\[
S_3 = \frac{1977 \text{ sales}}{1975 \text{ sales} + 1976 \text{ sales} + 1977 \text{ sales}}
\]

Using the data from calculation (1) the value of the weights are:

\[
S_1 = .185
\]

\[
S_2 = .370
\]

\[
S_3 = .445
\]

If there is a constant level of assessment the results are identical to those in (1):

\[
$1,000,000 \times .185 + $1,000,000 \times .370 + $1,000,000 \times .445
\]

\[
= \frac{5,000 + 10,000 + 12,000}{10,000 + 20,000 + 24,000}
\]

\[
= \frac{1,000,000}{50}\%
\]

\[
= 2,000,000
\]

Thus this method does not introduce distortions in cases where there is consistent assessment performance.

If the 1977 level of assessment changes, as it did in (2), the total adjusted assessed value is not affected:

\[1\] By assuming that the total assessed value if $1,000,000 in each year we are assuming no inflation and no additions to total property wealth through new construction or improvements on existing properties. The effects of inflation, new construction, and improvements are briefly mentioned on page 28 of the text. Additional documentation is available.
$1,000,000 \times 0.185 + \frac{\$1,000,000 \times 0.370 + \$1,500,000 \times 0.445}{61.1111\%} = \$2,000,000$

Using this recommended methodology, a change in assessment performance will not lead to an erroneous indication of the adjusted assessed value.
Prior to the 1976 study, data for commercial and industrial property were maintained separately, and average assessment/sales ratios were calculated for each category. Beginning with the 1976 study, however, the two property types were combined and only one set of average assessment/sales ratios was calculated for the combined category.

Combining commercial and industrial properties into one category is likely to result in a change in the adjusted assessed value and a corresponding change in aids whenever average assessment/sales ratios differ between the two property types. The direction of change cannot be predicted without detailed information. The direction will depend upon:

1. the relative magnitude of total industrial assessed value vs. total commercial assessed value;
2. the relative proportions of industrial versus commercial assessor’s market value in the sales ratio samples; and
3. whether a separate aggregate sales ratio for industrial property would be greater or less than the separate commercial aggregate ratio.

THE DISTORTION OF ADJUSTED ASSESSED VALUES

The following example demonstrates that commercial and industrial properties can be combined if they are comparably assessed. It is then demonstrated that distortions will occur if these properties are combined when their separate average assessment/sales ratios differ.

Suppose that a given town has $60,000 total assessed value in commercial property, and $40,000 in industrial property, and both types have aggregate assessment/sales ratios of 80 percent. Dividing each assessed value by 80 percent and summing the resulting adjusted assessed values yields a total adjusted assessed value of $125,000.
In this case, since both commercial and industrial property have the same assessment/sales ratio, the results are identical if the property types are initially combined, as in Table 12. Since either approach yields the same total adjusted assessed value, commercial and industrial properties can be combined into a single category if both have comparable assessment/sales ratios.

Now examine a different case where industrial and commercial properties are not comparably assessed. There are two commercial sales with the following individual assessment/sales ratios: $800/$1,000 = 80 percent and $400/$500 = 80 percent. The aggregate mean is ($800 + $400) ÷ ($1,000 + $500) = 80 percent. For industrial property there are also two sales, with the following aggregate mean: ($400 + $200) ÷ ($1,000 + $500) = 40 percent. If there is $70,000 of commercial total assessed value in the community and $50,000 of industrial total assessed value, then when commercial and industrial properties are handled separately, we obtain Table 13. Total adjusted assessed value is $212,500, the sum of the adjusted assessed values for commercial and industrial property. This is an accurate total.
If the two property types are combined into a single category, the combined aggregate mean is 60 percent \( \frac{([500 + 400 + 200] + [1000 + 500 + 1000 + 500])}{4} = 60\% \). This yields an adjusted assessed value of $200,000 (Table 14). In this case, the decision to combine the two property types into a single category results in an aggregate assessment/sales ratio which is not accurate for either property type (commercial is assessed at 80 percent, industrial at 40 percent), and a decrease in adjusted assessed value occurs. Since the adjusted assessed value has declined (from $212,500 to $200,000), the district would have to raise less taxes locally, and would receive more state aids.

### EFFECT OF SAMPLE COMPOSITION

Besides altering the adjusted assessed value, combining these property types when they are not comparably assessed will cause fluctuations in the adjusted assessed value over time. The adjusted assessed value will change as the relative proportions of industrial assessor's market value to commercial assessor's market value in the sales ratio sample change from year to year. One factor that might cause this is an unusual number of sales in a given year for one of the property types.

To illustrate, suppose that instead of two industrial sales, there are three. The previous two industrial sales were both as-
sessed at 40 percent; let the third industrial sale also have a 40 percent individual assessment/sales ratio, with an assessor's market value of $400 and a sale price of $1,000. The aggregate mean for industrial property is still 40 percent; therefore this additional sale leaves Table 13 (where commercial and industrial properties were handled separately) unchanged. The adjusted assessed value for industrial property is still $125,000, and the sum of commercial and industrial adjusted assessed values remains at $212,500. Hence there is no change in aids if the two property types are treated separately. However, when the property types are treated in a combined fashion, the aids will change simply because of the additional sale. Recalculating the combined aggregate mean with the additional sale included, we have: 

\[
\frac{\left(800 + 400 + 400 + 200 + 400\right) + \left(1,000 + 500 + 1,000 + 500 + 1,000\right)}{5} = \frac{2,200 + 4,000}{5} = 55\%.
\]

The calculation of the new adjusted assessed value is shown in Table 15.

<table>
<thead>
<tr>
<th>Property Type</th>
<th>Assessed Value</th>
<th>Aggregate Assessment/Sales Ratio</th>
<th>Adjusted Assessed Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Commercial and</td>
<td>$120,000</td>
<td>55%</td>
<td>$218,182</td>
</tr>
<tr>
<td>Industrial</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The correct adjusted assessed value is $212,500, as calculated in Table 13, where separate commercial and industrial categories are maintained. In Table 14, where a single category was first used, the adjusted assessed value was estimated to be $200,000, thus understating local property wealth and increasing state aid. With an additional industrial sale, the estimate of adjusted assessed value is $218,182. This district would now lose aids, rather than gaining by combining as in the earlier example.

Combining commercial and industrial properties when the two are not comparably assessed makes the combined aggregate ratio and the adjusted assessed value sensitive to sample composition. This has been illustrated by changing the volume of industrial sales. In the present example, if a great deal of industrial property sold, the overall aggregate mean would fall, and aids would fall. If an unusually high volume of commercial property sold, aids would tend to increase.

Furthermore, the use of appraisals would have the same effects. Whether the additional industrial observation is an individual assessment/sale ratio or an assessment/appraisal ratio does not effect the example. If a decision were made to enlarge the combined commercial and industrial sample with industrial appraisals which are comparable to industrial sales (i.e., they are good industrial appraisals) aids will decrease. If good commercial appraisals are performed, aids will increase.
APPENDIX D
AGRICULTURAL PROPERTY

The problems encountered with agricultural properties are similar to the difficulties which arise when commercial and industrial property are combined, despite differences in their average assessment/sales ratios. The solution for commercial and industrial properties is to separate the two categories, and by way of analogy a similar action might seem appropriate here. Unfortunately, it would be impossible to handle improved and unimproved agricultural land separately for assessment/sales ratio and school aid purposes. To do this, it would be necessary to have separate categories—improved vs. unimproved—maintained on the assessment rolls. This is impossible because the manner in which a parcel sells determines whether it is an improved or unimproved observation. For example, a sale of a 200 acre farm with buildings would fall into the improved category. If the new owner later decides to keep 100 acres with the homesite and buildings and sells 100 acres without buildings, this sale would now fall into the unimproved category. Since the assessor does not know what parcels will sell or how they will be divided for sale, separate categories cannot be maintained.

Even if this difficulty did not exist, separating improved from unimproved does not adequately address the problem. The distinction between these categories is that one contains only land while the other includes both land and buildings. If average assessment/sales ratios often differ as the Mann-Whitney tests indicate, the logical conclusion is that the level of assessment differs between land and buildings. If better indicators are to be obtained, the two current categories should be replaced by a separate land and a separate building category.

The two categories, land and buildings, are currently maintained by the assessor, and the limited and estimated market values for each category are generally recorded for each Certificate of Real Estate Value which is sent to the Department. Thus there is no problem in obtaining the appropriate assessor's values. The difficulty with implementing these categories occurs with sale prices—for improved sales (land and buildings) only the combined sale price is given.

POSSIBLE ALTERNATIVES

Although there are several possible procedures for dividing the sale price into separate considerations for land and buildings, or to obtain proxies for these sale prices, all approaches have major shortcomings. Better procedures may be costly to implement,
and/or it may be very difficult to evaluate the quality of the results.

One approach might use unimproved sales (land only) to develop average assessment/sales ratios and expected sale prices for agricultural land. If statistical tests suggest there is no apparent difference in land fertility between improved and unimproved sales in a given area, the expected sale price of the land can be calculated, and the building value could then be calculated as a residual; the value of buildings being the total sale price on the Certificate of Real Estate Value minus the expected sale price of the land.

An alternative, also an "appraisal-residual" technique, would be to appraise the buildings, either by on-site evaluation or through use of computerized multiple regression techniques. Using the appraisal as a proxy for the sale price of the buildings, the value of land can be calculated as a residual. In a practical setting, it may be advisable to select the approach which seems most applicable in a given situation. For instance, if the sale under consideration is the sale of a very small farm, valuing the land and calculating the value of buildings may be acceptable. If there is little land, a sizable error in land valuation may not significantly influence the accuracy of the estimated building value. On the other hand, if this approach were used for a large farm, a slight error per acre in the value of land might yield an extremely poor estimate for the building value. In such a case it might be best to appraise the buildings and calculate land value as a residual.

A third approach might use unimproved sales to develop average assessment/sales ratios for land. Building values would not be determined as a residual from the total sale price; rather, a computerized multiple regression approach could be used to estimate building value based upon building characteristics, farm size, location, land fertility, crop prices, and other factors. This

---

1 The simplest approach would be to ask the individuals who file the certificate to decompose the sale price into the consideration for land and the consideration for buildings. However this approach may yield substantial inaccuracies. The individuals may not take the necessary time to accurately answer these questions, they may not know the answers, or they may be influenced by their suspicions concerning the use of this information. Suppose the individuals feel that the information they provide will influence the future limited market values (and hence taxes) on this property. They also feel that the consideration for the building will be closely reflected in future limited values, but the consideration for land will not strongly influence future limited values. In this case there would be a tendency to understate the consideration for the buildings. Depending upon opinions of individual buyers, an opposite bias is also possible.

2 Even if fertility differentials do exist, it should be possible to estimate the dollar values of these differentials.
approach would not have the problem inherent in the first two approaches, where an erroneously high land value may automatically lead to an erroneously low building value, and vice versa. A further possibility is to adjust appraisals through use of sales information. Suppose a farm has sold for $100,000, and a separate land appraisal and building appraisal are performed. The appraisals value the buildings at $40,000 and the land at $70,000, which gives a total of $110,000. This total is 10 percent higher than the actual purchase price. A ratio of total sale price to total appraised value could be formed and used as a correction factor to make the separate appraised values comparable to the total sales price. Multiplying the separate appraised values by the correction factor, we have for buildings $40,000 x ($100,000 ÷ $110,000) = $36,364, and for land $70,000 x ($100,000 ÷ $110,000) = $63,636. The value for buildings and the value for land now sum to the total price ($36,364 + $63,636 = $100,000). The following assumptions are implicit:

(1) Sale prices are the fundamental measure of value. Appraisals should therefore be made as consistent as possible with sale prices.

(2) Both the land and building appraisals can properly be subjected to the same correction factor (i.e., it is assumed that they err by the same proportion).

DIFFICULTIES WITH ALTERNATIVE PROCEDURES

As previously mentioned, these approaches may be difficult to implement and evaluate. Furthermore, they may be incompatible with recent statutes.

With appraisal-residual approaches, land appraisals can be constructed or estimated from unimproved land sales, given that all important factors which may influence value are included in the analysis. (Differences in fertility; size, location, and other factors must be incorporated.) The accuracy of these appraisals cannot be determined satisfactorily; one would have to accept the estimates as given, granting them validity since they are the best indicators obtainable given time, money, and staffing considerations. The building value is just as troublesome, whether estimated directly or as a residual. Since buildings sell with some land included, improved sales cannot be directly compared to building appraisals. One possibility is to estimate statistically the value of the buildings exclusive of the land. However, as with land valuation, the accuracy of the approach is difficult to determine. A possibility is to compare the statistically determined building values with on-site building appraisals of the same properties. In this case one would be comparing one estimate with another.
In addition, separate land and building categories may be costly to implement and maintain. Given the uncertainty of the benefits, the new categories may not be feasible.

The suggestions contained in this appendix may be inconsistent with the assessment/sales ratio dispersion penalty, \(^1\) instituted in Minnesota Statutes, 1977 Supplement, section 477 A.04. If agricultural property is subject to an assessment/appraisal ratio study, while other property types are basically subject to an assessment/sales ratio study, two questions must be answered:

1. Would the dispersion in assessment/appraisal ratios for land and building categories be comparable to the dispersion in assessment/sales ratios for these categories, assuming they could be observed?

2. If the dispersion is not comparable, is it justifiable to levy a penalty based upon assessment/appraisal ratios?

In the forms suggested, the appraisal approaches are inconsistent with Minnesota Statutes, 1977 Supplement, Section 124.212, Subdivision 10, which will influence the handling of agricultural properties beginning with the 1977 assessment/sales ratio studies. This statute provides that agricultural "sale prices" used for the sales ratio studies shall be an arithmetic average of sale price and the value of agricultural land based upon crop yields.

The new law may affect the findings in Chapter IV and the alternative procedures outlined here to separate land and building values. When the law is implemented, Mann-Whitney tests may suggest that buildings and land are comparably assessed. Since averaging estimated land value with sale prices may tend to lower the "averaged" sale prices used in the study, the sales ratios for land may increase. In such cases, the present categories of improved and unimproved land could be maintained. In cases where there continue to be differences in assessment between land and buildings, the most desirable course of action is uncertain. Modifying the methods suggested in this chapter to gain consistency with the intent of the new law will increase the difficulty of determining both the accuracy of the new procedures and their benefits and costs.

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\(^1\) Beginning in 1980, this provision would penalize any assessment district where the coefficient of dispersion for assessment/sales ratios is more than 10\%. The penalty increases as the coefficient of dispersion increases, with a one dollar per capita penalty for coefficients of dispersions between 10\% and 12.5\%, increasing to a five dollar per capita penalty for coefficients of dispersion greater than 15\%. These penalties would be deducted from the local government aid allocated to the district.
A conceptual problem arises as well. Using an average of sale price and value based upon agricultural pursuits is a compromise between two methods of valuing property, and thus a separate land assessment/sales ratio based upon this averaging would also be a compromise. Such a practice will introduce subjectivity and uncertainty into the determination of assessment/sales ratios, and the validity of the ratios will become very difficult to ascertain. Also, if assessment/sales ratios (or assessment/appraisal ratios) are maintained for land and buildings separately, similar problems and questions arise for agricultural buildings. It must be decided whether the assessment/sales ratios for buildings should be based upon a single valuation system, or a combination of two approaches.
APPENDIX E
WRITTEN RESPONSES OF THE MINNESOTA
DEPARTMENT OF REVENUE
July 7, 1978

Mr. Bruce Spitz  
Deputy Legislative Auditor  
Legislative Audit Commission  
Program Evaluation Division  
Veterans Service Building  
St. Paul, Minnesota 55155

Dear Mr. Spitz:

I have enclosed a reply to the report you prepared concerning the Department of Revenue's assessment/sales ratio study.

In general, I agree with most of the recommendations contained in the report. However, the recommendations do raise some serious questions that must be answered before any changes can be made in how the study is conducted.

I commend you and Ed Burek for the excellent work you both did in preparing the report. Also, I appreciate the cooperative manner in which the entire study was done.

Sincerely,

[Signature]

ARTHUR C. ROEMER  
Commissioner of Revenue

ACR:DJE:kjz
RECOMMENDATION:

(1) All sales (or appraisals) should be matched with assessor's market values in the year of sale (or appraisal). This is the fundamental step which must be taken if any significant improvement is to occur in the sales ratio studies.

The Department of Revenue agrees with the findings of the Legislative Audit Commission, but with certain reservations about the recommendations. While it is true that the proposed matching of data on a year for year basis will yield statistics which are individually more precise and collectively more valid, this procedure does introduce a number of serious practical problems which must be dealt with before any changes in processes are contemplated.

1. Such a procedure automatically eliminates from consideration for the Sales Ratio Study any properties involving new construction or any types of physical change between the assessment date and the date of the sale. The effects of this will be particularly inequitable in those communities that have a large amount of new home construction, remodeling or similar change. This is due to the fact that the assessment sales ratio will not in fact be representative of the assessment level of the community as a whole based on all sales but only for those older homes or unimproved homes which sold during the study period. It can be demonstrated that there are significant differences in the assessment level of older homes versus that of new construction; hence any study which automatically excludes either class will give a distorted picture of the overall assessment level of the community.

2. The recommendation of the Legislative Audit Commission suggests that the problems of using year to year matching which relate to the computation of adjusted assessed values can be overcome by following
a related recommendation, which is dependent upon a three year average of assessed values by property type. The Department believes that the use of an "average" assessed value, whatever weighting factor may be applied, would give a more distorted and confusing picture of the actual tax-generating capability of a school district than does the present method.

3. If any change from the present methodology is contemplated, it must be noted that there will be significant administrative problems involved during the transition period. Since it is impossible to get identical samples for studies conducted both under the existing system and the proposed system, truly parallel studies cannot be carried out and the precise implications, both direction and scale of the change difficult to determine. Any additional data required for the new system would have to be gathered, screened and analyzed before any valid data can be obtained. This would require during the initial stages a significantly greater dedication of funds, personnel and time than is presently available.

Additional problems relating to the assessed value weighting proposal which are closely related to this recommendation will be discussed further below.

RECOMMENDATION:

(1) As recommended previously, all observations should be matched with assessor's market values in the year of sale. This will permit the following changes which improve the quality of the screening procedures:

(a) Only the newest year's data would need to be screened. The amount of data to be scrutinized annually will decrease to approximately one-third its current level.
(b) The present practice of updating older sales can be eliminated. Eliminating this phase will permit data editing to begin earlier, and to be performed more intensively.

The Department of Revenue agrees with the findings and recommendations of the Legislative Audit Commission. After the year by year data matching system had been in operation for a sufficient period of time, there should be significant savings in time and effort devoted to screening old samples and a corresponding increase in the amount of additional attention that could be given to new sample items. This would also eliminate an element of clerical error which will inevitably appear during the manual or automated processing of the study. As indicated elsewhere, there would be other problems involving new or improved properties, as well as a significant transition problem.

RECOMMENDATION:

(1) Given adoption of the data matching recommendation, to calculate adjusted assessed values a sales-value-weighted average of total assessed value should be divided by the appropriate aggregate assessment/sales ratio for each property type and municipality. The number of years of total assessed values used in the calculation should match the number of years used in calculating the aggregate assessment/sales ratio.

The Department of Revenue acknowledges that the present market value method of weighting ratios for use in determining adjusted assessed values may contain some distortion based upon the classification ratios assigned to various types of property. However, the proposed system for computing adjusted assessed values would also introduce certain complication in the following areas:

1. An assessed value weighting system will place greatest emphasis on the assessed value for those property types which are represented by the smallest sample size, i.e. commercial and industrial properties, which are assessed at a higher percentage of market value than residential and farm areas. The effects of this change in procedure will be felt most strongly in those urban and metropolitan areas where commercial and industrial value has a relatively higher
percentage of value than in agricultural or suburban communities.

2. The use of the data matching system suggested in the first recommendation will have a doubly punishing effect on assessors who have made a determined effort to increase the level of assessment in their jurisdictions. On the one hand the increase in their assessment level will be reflected only in the most recent year's sales ratios. On the other hand the increased assessed value due to this reassessment would take effect immediately in the total assessed value in the school district. The Legislative Audit Commission recommendation proposes to resolve this problem by adopting a three year sales-weighted average of assessed values. There seem to be three problems inherent in this approach.

A. In spite of the sales-weighting techniques distortions due to differences in sample size from year to year will be present. For a community which had a significant number of sales in a property type in the first year of the study and where new construction on a significant scale took place in the second and third year of the study, together with a reassessment, the amount of the improvements and the new, higher assessment level would show in the total assessed value in the school district but would not be wholly reflected in the sales ratio for that district.

B. The adoption of an average assessed value concept removes the determination of the adjusted assessed value one step further from the current actual tax producing capability of the school district.
C. There are a number of additional practical problems which must be resolved probably through new legislation which involve such matters as the 8% limitation on increases in adjusted assessed values, the effects of mineral, timber, public utility, personal property and problems involving splits or annexations between and within school districts and related problems involving the determination of sales ratios for local government aid purposes.

RECOMMENDATION:

1. The Department of Revenue should hire additional personnel to implement statistical testing procedures relating to the design and uses of assessment/sales ratio studies.

The Department of Revenue is in agreement with the recommendation of the Legislative Audit Commission that there is a need for additional personnel with expertise in advanced statistical testing procedures to assist Revenue staff in evaluating the nature and scope of any changes to the study procedures. Such a staff increase is not currently provided for departmental budgets nor are such personnel currently available within the Department with the time or background to deal with these problems.

RECOMMENDATIONS:

1. The Department of Revenue should develop and implement a test procedure to determine the advisability of using a particular set of appraisals. An alternative, the use of a fourth year of sales, should also be tested. A set of guidelines for testing appears in Chapter IV of the report.

The Department of Revenue is in agreement with the Legislative Audit Commission in its recommendations that appropriate tests should be run to determine the size of sample necessary to produce a reliable sales ratio for a given jurisdiction. This recommendation would be adopted in conjunction with
the previous recommendation that the Department hire additional personnel versed in statistical methods. Failure to conduct such tests in the past has been the result of a lack of time and expertise rather than an unwillingness to make the determinations involved.

Although there is no disagreement with the recommendations of the Legislative Audit Commission, the findings upon which this recommendation were based need to be examined in greater detail. There are three particular areas of concern which need to be brought up in this regard.

1. Both the Mann-Whitney and Kruskal-Wallis tests may be proving what is not in contest; that is, that the sales ratios for commercial appraisals are different than those for commercial sales in our study. This is not surprising in view of the fact that those commercial properties which sell tend by nature to be "dogs" and the samples which are appraised tend to be going concerns which are not put on the market. Thus, the use of appraisals is not intended solely to expand an existing substratum of this commercial sample, but rather to open up an entirely different potential sample substratum for investigation.

2. Aside from the advantages to be gained by studying those commercial properties not currently on the market in developing a sales ratio, there is the obvious necessity of obtaining a sufficient sampling in areas with very limited commercial activity. If we were to eliminate appraisals which might not produce positive results on the Mann-Whitney or Kruskal-Wallis tests (or which cannot be tested) we would be faced with the prospect of generating ratios, even on a county wide basis, based on an extremely small number of samples. It is not felt that this would be a significant improvement over the merging of sales and appraisals under the present system. The use of a small sample - a very real problem under either system - would be further exacerbated
by deleting appraisals from the study. This could tend to discriminate against specific communities or counties, where the sales sample would be limited and the effects of each individual sample item therefore, accentuated.

3. The sample used by Legislative Audit Commission included a number of commercial, industrial and apartment appraisals which were admittedly questionable quality. These included a number of samples where supporting documentation was non-existent or which were in fact old appraisals which had been periodically updated. It is our belief that the appraisals in use in the current study which were dated 1976 and later are substantially better in quality than those which constituted the bulk of the Legislative Audit Commission sample. Further study will be necessary to determine whether a system of mass appraisals would be more statistically significant in estimating the assessment sales ratio for commercial property than a limited number of the precise appraisals as is currently in use.

4. The Department does not feel that the suggested alternative of using a fourth year of sales would be appropriate for our study. For practical reasons we feel it is advisable to treat all areas of the state as uniformly as possible. To selectively increase the size of our sample by adding an extra year of sales (or to reduce it by selecting only a limited number of sales from a larger sample) would be an unrealistic solution.
RECOMMENDATIONS:

(1) Since even conservative statistical tests suggest that commercial and industrial properties cannot validly be merged for many counties, these categories should generally be maintained and processed separately for assessment/sales ratio and aid calculation purposes. If commercial and industrial properties are comparably assessed, sales ratios and aids will be identical whether they are combined or handled separately. If they are not comparably assessed, combining the two categories will produce assessment/sales ratios which are not accurate for either property type and will result in distorted aid allocations.

The Department of Revenue believes that the combination of commercial and industrial properties into one sample is a reasonable decision. While the tests conducted by the Legislative Audit Commission would appear to indicate possible problems which may arise from merging the samples in the potential problems from not combining the sample would be even greater.

1. The distinction between commercial and industrial properties for assessment purposes is a very gray area. Similar properties in different counties or even in different areas within a county will be classified differently. Thus separating the sample would lead to comparison of unlike properties in different counties.

2. The industrial sample in our study, even on a county wide basis, would be non-existent in many counties and extremely small in virtually all counties of the state. This would require us to determine a ratio based on a very small sample for industrial property, which may or may not have an adverse effect on those areas with a large amount of value identified by the assessors as industrial but for which there is a small sample. The complication comes in identifying the effects in individual instances.

We will acknowledge the necessity for further study in this area.
FINDINGS:

(1) Based upon statistical tests, in nearly half the counties in the state there is evidence that improved agricultural sales and unimproved sales have different average assessment/sales ratios.

(2) The difference between the assessment/sales ratios for the two categories appear to be due to differences in assessment levels between land and buildings.

The Department of Revenue agrees in principle with the findings of the Legislative Audit Commission that the current method of treating agricultural property, i.e. combining the improved and unimproved farm properties under the general heading of Agricultural Properties, may in many cases be inadequate as a means of determining the actual assessment level of agricultural property. However, we, like the Legislative Audit Commission, have not been able to arrive at a feasible alternative to the present procedure.