Regression Analysis Methods and Limitations

APPENDIX B

As Chapter 2 explained, our analysis showed that students who qualified for subsidized lunch represented an acceptable but limited proxy for students performing below academic standards. We concluded that the proxy is limited because not every student qualifying for free or reduced-price lunch achieved below academic standards. We defined academic standards using results from the Minnesota Comprehensive Assessments (MCAs), Minnesota's required statewide tests (briefly explained at right).

Minnesota Comprehensive Assessments

MCAs measure student achievement of academic, grade-level standards in three subject areas: mathematics, reading, and science.

Students' scores on the tests fall into one of four achievement levels:

(1) exceeds achievement standards, (2) meets achievement standards, (3) partially meets achievement standards, or (4) does not meet achievement standards.

— Minnesota Department of Education

We conducted our analysis to better understand the statistical relationship, or correlation, between two groups of students—those who are not meeting academic standards and those who come from low-income families. This is important because compensatory education revenue is intended to help students who are not meeting academic standards appropriate for their age group. However, the amount of compensatory education revenue a school district receives depends in part on a different set of students—those who come from low-income families. For the purpose of compensatory education revenue, students who qualify for free or reduced-price lunch are considered to be low income.

Defining Correlation

A **correlation** is a statistical measure of the extent to which two or more variables are connected or fluctuate together. Correlated variables occur together in a way that is unexpected if by chance alone.

— Merriam-Webster

A common statistical procedure to determine a correlation between two groups (or variables) is a "multiple regression analysis." With this analysis, we could estimate the likelihood of a correlation between students qualifying for subsidized lunch and a tendency to not meet academic standards. A multiple regression analysis is useful in that it can account for other possible explanations of poor MCA results and isolate a single factor to explain the tendency for those results.

Our analysis occurred in three phases—selecting data inputs, deciding on and using statistical methods, and reporting results. In the rest of this appendix, we summarize the three phases and discuss the limitations of our analysis.

Data Inputs

The first phase of our analysis was to select the data and variables. We obtained from the Minnesota Department of Education (MDE) individual, student-level data for every public school student that took the MCAs from fiscal years 2011 through 2018. A student may have taken more than one MCA in a subject area during this time period, sometimes resulting in more than one observation per student by subject. Individual student data provided a large number of observations, allowing us to estimate more precisely the likelihood of a correlation.[1]

We used three datasets—one for each MCA subject area of math, reading, and science. Included in each dataset was the "outcome" (or dependent) variable, which was a student's MCA achievement level. Datasets also included the independent variable of most interest to us, which was whether a student qualified for subsidized lunch. Using a multiple regression, these two variables allowed us to analyze the correlation between students who did not meet academic standards and students who qualified for subsidized lunch.[2]

> Two types of variables are in a regression analysis: the independent variable and the outcome (or the dependent) variable. The independent variable is an input in an equation, and its value is usually taken as given. The outcome variable is calculated in an equation with independent variables. The value of the outcome variable is, therefore, dependent upon the independent variables.
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> **— Universal Class, "Understanding Variable Use in Algebra"**

To more easily describe our variables, we classified them into four types—individual student-level, school- or district-level, school revenue, and other. Below is a brief summary of the classifications.

- **Individual student-level variables.** These variables accounted for individual student circumstances that may affect the likelihood of a student meeting MCA standards. Such variables included race, homelessness, certain grade levels, enrollment in special education, and whether the student qualified for free or reduced-price lunch.

  The key variable of interest was whether the student qualified for free or reduced-price lunch. If the estimated coefficient to this variable was negative and statistically significant, then students that qualified for free or reduced-price lunch were less likely to meet academic performance standards.

[1] The multiple regressions selected for this analysis produce consistent estimates such that, as the number of observations increase, the estimates approach the true value for the population. Consequently, standardized test results have the most value when viewed across a large number of students taking the tests.

[2] The total number of observations was 3.3 million for the math assessment, 2.6 million for reading, and 1.3 million for science. A main reason why there were fewer observations in the reading data set compared with math is that schools administered the MCA on reading for two fewer years in the period we analyzed. For science, schools administered the MCA to students in grades 5, 8, and one high school grade, which is four fewer grades than the MCAs for math or reading.

- **School-level and district-level variables.** To account for the correlation between the school environment and MCA achievement levels, we included district-level variables such as "student-teacher ratios" and "graduation rates." We also included a school-level variable on "percentage of students qualifying for free or reduced-price lunch." This variable captured the correlation with student achievement levels due to concentration within a school building of students qualifying for free or reduced-price lunch. (It was separate from the individual-level variable of whether an individual student qualified for the subsidized lunch program.) In addition, binary variables defining regions captured differences in student performance across geographic regions.[3]

- **School revenue variables.** Several school revenue variables accounted for the financial resources available to help students achieve. The variables we included were compensatory education revenue, English learner revenue, and other general education revenue. We also included adjusted net tax capacity, which represents the property value used for calculating most local school taxes.

Methodology

The second phase of the analysis was to select and use regression methods. We selected a model that could estimate the probability of a student meeting MCA standards while including additional variables intended to isolate other confounding factors, such as the correlation of homelessness with lower academic achievement.

The goal of the analysis was to analyze the appropriateness of using students qualifying for free or reduced-price lunch as a proxy for students who were not meeting MCA academic standards. More specifically, we looked for evidence on whether an increase in the number of students qualifying for subsidized lunch generally corresponded to an increase in the number of students failing to meet academic standards.

Besides analyzing correlations using our primary approach, we investigated whether alternative statistical approaches would produce opposing results. That is, we undertook a number of alternate methods to test whether we would find a tendency for students who qualified for subsidized lunch to fail disproportionately to meet academic standards. Below, we describe the primary model in our analysis and then explain the alternate approaches we used to test our results.

The Primary Model

The first model we used to estimate tendencies is a "probit" regression, which allowed us to estimate the probability of a student failing to meet academic standards. (See a brief description at right.) The outcome variable was whether a student met academic standards. We included more than 45 independent variables, including whether

A **probit regression model** estimates the probability that an outcome occurs, given a set of independent variables. It is useful when the outcome variable of interest has two possible outcomes, for example, meeting academic standards or not meeting them.

— Statistics How To

[3] A binary variable equals one (1) when the observation fits the category and otherwise equals zero (0).

the student qualified for subsidized lunch. Exhibit B.1 on page 6 lists them. We ran the probit regression three times, estimating separate probabilities on whether students failed to meet standards in the three subject areas of math, reading, and science.

Alternative Statistical Approaches

In addition to our primary model, we used different statistical approaches to estimate correlations. In one approach, we changed the definition of what it meant to "not meet standards." The purpose was to determine whether qualifying for free or reduced-price lunch still correlated with meeting standards under the alternate definition.

We also constructed variables in different ways to test whether the differences would produce differing results. As an example, for the variable on compensatory revenue per student, we substituted the dollar amount of total revenue per school district or charter school. None of the different approaches affected the correlation with free or reduced-price lunch. Results of our alternative approaches start on page 13 of this appendix.

Further, we conducted three alternative regressions, which we explain next.

- **Random Effects Probit Regression.** Because each student is unique, we included a random element that was specific to each individual student. This helps when generalizing the results to new students who are not yet in the dataset. The random effects regression model is different from a standard probit regression, which has an error term only for each student taking a test in a particular subject and on a particular day. In contrast, the random effects probit regression included a second error term for each student, regardless of the day a student took the test.

- **Ordered Probit Regression.** We used an ordered probit model to more exactly estimate ordered, categorical outcomes. The ordered probit regression allowed us to examine the correlation among MCAs' four ranked achievement levels: exceeds standards, meets standards, partially meets standards, or does not meet standards. The ordered probit regression analyzed separately each of the four achievement levels. By contrast, our primary probit model had grouped the four achievement levels together into two groups. Consequently, with the ordered probit regression, we could be more exact about whether qualifying for free or reduced-price lunch was correlated with lower achievement levels.

- **Bivariate Probit Regression.** We also examined whether the choice to apply for subsidized lunch might itself be an outcome variable. Qualifying for free or reduced-price lunch depends upon a family's income, but not every family chooses to apply. This gave us two outcome variables to analyze: (1) whether students met academic standards and (2) whether students qualified for subsidized lunch. The two outcomes appeared correlated, indicating the need to use a bivariate probit model because, in general, that model estimates outcomes that are interrelated. The bivariate probit model simultaneously estimated the outcomes of whether a student met academic standards and qualified for subsidized lunch.

Outcomes

The results of our analyses indicated a tendency that students qualifying for subsidized lunch were less likely than others to meet academic standards. Furthermore, changes we made to our primary regression to adjust for possible statistical issues with the data made no difference in our conclusion.

Before describing our regression results, we list in Exhibit B.1 information on variables we used in our analysis. With the exception of the first variable ("Meets academic standards"), the variables in Exhibit B.1 are independent variables that represent other possible explanations of MCA achievement levels.

The exhibit includes the mean (average) and "standard deviation" of each variable in OLA's regression. An average is the central or typical value for a variable and equals the sum of all the values of the variable divided by the number of observations. For example, under the math assessment in Exhibit B.1, the first variable "Meets academic standards" has an average of 0.81, indicating that students met the academic standards in math 81 percent of the time.[4] Conversely, 19 percent of the students did not meet academic standards when taking the math assessment.

The standard deviation is a measure of the variation, or dispersion, of values for a variable. With a given average, a higher standard deviation indicates a more dispersed set of values for the variable. For the first variable "Meets academic standards" under the math assessment in Exhibit B.1, the standard deviation was a fairly wide measure of dispersion at 0.3899.

[4] In this case, meeting academic standards was defined to include students who "exceeded" MCA standards, "met" the standards, or "partially met" the standards. As described later in this appendix, we defined "meeting academic standards" in two separate ways to test a correlation between students qualifying for subsidized lunch and meeting the standards.

Exhibit B.1: Means and Standard Deviations of Variables, by MCA Math, Reading, and Science Datasets, Fiscal Years 2011 through 2018

	Math		Reading		Science	
	Mean	Standard Deviation	Mean	Standard Deviation	Mean	Standard Deviation
Meets academic standards[a, b]	0.8130	0.3899	0.7876	0.4090	0.7821	0.4128
Qualifies for free or reduced-price lunch[a]	0.3790	0.4851	0.3797	0.4853	0.3570	0.4791
3rd grade[a, c]	0.1537	0.3606	0.1453	0.3524	0	0
4th grade[a, c]	0.1533	0.3603	0.1450	0.3521	0	0
5th grade[a, c]	0.1524	0.3594	0.1438	0.3509	0.3458	0.4756
6th grade[a, c]	0.1513	0.3583	0.1428	0.3499	0	0
7th grade[a, c]	0.1514	0.3584	0.1428	0.3498	0	0
8th grade[a, c]	0.1503	0.3573	0.1416	0.3487	0.3383	0.4731
9th grade[a, c]	0	0	0	0	0.0241	0.1534
10th grade[a, c]	0	0	0.1387	0.3456	0.2400	0.4271
11th grade[a, c]	0.0877	0.2828	0	0	0.0456	0.2087
American Indian[a]	0.0234	0.1512	0.0239	0.1527	0.0220	0.1465
Asian American[a]	0.0723	0.2590	0.0735	0.2609	0.0711	0.2571
Hispanic[a]	0.0811	0.2730	0.0837	0.2770	0.0768	0.2663
Black[a]	0.1105	0.3136	0.1136	0.3174	0.1037	0.3049
Female[a]	0.4886	0.4999	0.4889	0.4999	0.4886	0.4999
Receives special education[a]	0.1393	0.3463	0.1389	0.3459	0.1354	0.3422
Homeless[a]	0.0079	0.0884	0.0085	0.0919	0.0070	0.0833
Primary language other than English[a]	0.1409	0.3479	0.1440	0.3511	0.1349	0.3416
Pilot district[a]	0.1304	0.3368	0.1303	0.3366	0.1313	0.3378
Alternative Learning Center and Programs[a]	0.0069	0.0827	0.0066	0.0809	0.0095	0.0971
Student assessed in Fiscal Year 2013[a]	0.1123	0.3158	0.1631	0.3695	0.1390	0.3459
Student assessed in Fiscal Year 2014[a]	0.1314	0.3378	0.1641	0.3704	0.1409	0.3479
Student assessed in Fiscal Year 2015[a]	0.1321	0.3386	0.1654	0.3715	0.1439	0.3510
Student assessed in Fiscal Year 2016[a]	0.1329	0.3395	0.1672	0.3732	0.1428	0.3499
Student assessed in Fiscal Year 2017[a]	0.1347	0.3414	0.1694	0.3751	0.1458	0.3529
Student assessed in Fiscal Year 2018[a]	0.1361	0.3428	0.1708	0.3763	0.1481	0.3552
Graduation rate	0.8397	0.2014	0.8493	0.1990	0.8572	0.1782
Dropout rate	0.4457	1.8429	0.0570	0.0519	0.0570	0.0526
Attendance rate	0.9455	0.0184	0.9448	0.0188	0.9453	0.0193
Student-to-teacher ratio	15.6912	1.9848	15.6043	1.9740	15.6834	1.9941
Student-to-classroom-aide ratio	115.8255	142.9090	113.9909	144.4000	113.8916	122.9988
Percentage of students qualifying for free or reduced-price lunch (per student in the school)	38.0235	22.0269	37.9031	22.0468	36.1083	21.2709
Net tax capacity per student in school districts[d, e]	0.0076	0.0048	0.0077	0.0049	0.0077	0.0048
Compensatory revenue per student in the school[e, f]	0.6298	1.3279	0.6411	1.4338	0.5819	1.4241
General education revenue per student in the district[e, f]	8.5988	2.3647	8.6203	2.3937	8.6353	2.2597

Continued on next page.

Exhibit B.1: Means and Standard Deviations of Variables, by MCA Math, Reading, and Science Datasets, Fiscal Years 2011 through 2018 (continued)

	Math		Reading		Science	
	Mean	Standard Deviation	Mean	Standard Deviation	Mean	Standard Deviation
English learner revenue per English learner student in the district[e, f]	1.6306	8.0224	1.6699	8.7782	1.6456	8.4674
Number of English learners per student in the district	0.0607	0.0864	0.0626	0.0881	0.0580	0.0797
Average daily membership[g]	10.2268	11.6743	10.2345	11.6717	10.0185	11.4319
Charter school[a]	0.0552	0.2283	0.0572	0.2323	0.0488	0.2155
Cooperative[a]	0.0054	0.0735	0.0049	0.0697	0.0059	0.0768
Student attended school in a rural county[a, h]	0.2160	0.4115	0.2156	0.4113	0.2206	0.4147
Student attended school in Hennepin or Ramsey county[a]	0.2861	0.4520	0.2870	0.4523	0.2775	0.4478
Student attended school in a suburban county[a, i]	0.2586	0.4378	0.2581	0.4376	0.2605	0.4389
Student attended school in a smaller metropolitan statistical area[a, j]	0.1509	0.3579	0.1513	0.3583	0.1522	0.3592
Previous achievement level was "does not meet standards"[a]	0.1058	0.3076	0.1198	0.3248	0.0044	0.0659
Previous achievement level was "partially meets standards"[a]	0.1315	0.3380	0.1093	0.3120	0.0049	0.0702
Previous achievement level was "meets standards"[a]	0.2388	0.4264	0.2380	0.4259	0.0079	0.0884
Previous achievement level was "exceeds standards"[a]	0.1641	0.3704	0.1092	0.3119	0.0034	0.0584

[a] Binary variable where the value equals one (1) when the observation fits the category and otherwise equals zero (0).

[b] Dependent variable for which the standards are those measured by students' achievement levels on the Minnesota Comprehensive Assessments (MCAs), the state's standardized tests. We defined meeting standards in two separate ways for our analyses. In this exhibit, meeting standards includes students who "exceeded" MCA standards, "met" the standards, or "partially met" the standards.

[c] Represents a student's grade level when taking the Minnesota Comprehensive Assessments (MCAs).

[d] In millions of dollars.

[e] Amounts from different years have been adjusted for inflation in Fiscal Year 2018 dollars.

[f] In thousands of dollars.

[g] In thousands of students.

[h] OLA defined as Minnesota counties that were not part of a metropolitan statistical area, according to the U.S. Office of Management and Budget as of 2013.

[i] OLA defined as Anoka, Carver, Dakota, Scott, and Washington counties. In estimating coefficients on Minnesota regions, the analysis did not include a separate variable for students living in the exurban counties of Chisago, Isanti, Le Sueur, Mille Lacs, Sherburne, Sibley, and Wright. These exurban counties were part of the Minneapolis-St. Paul-Bloomington metropolitan statistical area, according to the U.S. Office of Management and Budget in 2013. We could predict the exurban counties using the other county groups listed in this exhibit.

[j] OLA defined as counties located in metropolitan statistical areas (MSAs), according to the U.S. Office of Management and Budget as of 2013, outside the Minneapolis-St. Paul-Bloomington MSA. They were Carlton and St. Louis counties (Duluth MSA); Clay County (Fargo, ND, MSA); Polk County (Grand Forks, ND, MSA); Houston County (La Crosse, WI, MSA); Blue Earth and Nicollet counties (Mankato-North Mankato MSA); Dodge, Fillmore, Olmsted, and Wabasha counties (Rochester MSA); and Benton and Stearns counties (St. Cloud MSA).

SOURCES: Office of the Legislative Auditor, analysis of data from the Minnesota Department of Education; U.S. Bureau of Labor Statistics, *Consumer Price Index*, https://data.bls.gov/pdq/SurveyOutputServlet, accessed on December 23, 2019; and Executive Office of the President, U.S. Office of Management and Budget, *Revised Definitions of Metropolitan Statistical Areas, Micropolitan Statistical Areas, and Combined Statistical Areas, and Guidance on Uses of the Definitions of These Areas,* OMB Bulletin No. 13-01, February 28, 2013.

Primary Model Results

The tendency for students who qualify for subsidized lunch to be less likely to meet academic standards is not surprising given other literature on low-income students. Studies from elsewhere in the nation suggest that students from lower-income families tend to perform worse academically than higher-income students.[5] The results of our analysis indicate that this relationship is also true in Minnesota.

Moreover, it is very unlikely that the estimated negative tendency was due to random error in the data. The probability that no correlation exists between subsidized lunch and not meeting academic standards was less than 1 in 10,000.

Using our primary model, we estimated how many new students would not meet standards if they qualified for subsidized lunch. Based on our estimates, if 100 new students entered public schools in Minnesota, and all qualified for subsidized lunch, we would expect 6 more of these students to not meet standards in math, relative to the number of students if none had qualified for subsidized lunch. Similarly, we would expect 7 more students taking the reading assessment, and 10 more students taking the science assessment, to not meet academic standards in those subjects, respectively.[6]

Results from our primary model are in Exhibit B.2. The table shows the estimated coefficients (defined at right) resulting from our primary model. In general, a coefficient greater than zero indicates that a higher value for the variable correlates with higher achievement levels. If the value corresponding to the coefficient is less than zero, then the higher the variable's value, the lower the achievement level. For binary variables, a value greater than zero for the estimated coefficient indicates higher achievement levels when the student has the variable's characteristic or is enrolled in the program that the variable reflects. A negative coefficient indicates lower achievement levels. For example, an estimated -0.3 coefficient for the variable "Qualifies for free or reduced-price lunch" in Exhibit B.2 indicates that a student who qualifies for subsidized lunch is less likely to meet academic standards.

An estimated **coefficient** is a numerical measure. A positive sign on the coefficient indicates whether the probability will increase with a one-unit increase in the independent variable; a negative sign indicates the probability will decrease. The larger the coefficient's absolute value, the larger the probability, if all other independent variables are held constant.

— UCLA, Institute for Digital Research & Education, Statistical Consulting

The standard error indicates the uncertainty around the estimated coefficient. The smaller the standard error, the more precise the estimate and the lower the uncertainty about the estimated coefficient.

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[5] For example, see Sean F. Reardon, "The Widening Academic Achievement Gap Between the Rich and the Poor: New Evidence and Possible Explanations," in *Whither Opportunity? Rising Inequality, Schools, and Children's Life Chances*, eds. Greg J. Duncan and Richard J. Murnane (New York: Russell Sage Foundation, 2011), 91-116.

[6] We took the difference in average probabilities between all students qualifying for subsidized lunch and no student qualifying for subsidized lunch. We then multiplied that number by 100 to estimate how many additional students would not meet academic standards.

Exhibit B.2: Estimated Coefficients and Standard Errors from Primary Regression Model, Fiscal Years 2011 through 2018

	Math		Reading		Science	
	Coefficient	Standard Error	Coefficient	Standard Error	Coefficient	Standard Error
Qualifies for free or reduced-price lunch[a]	-0.3377***	0.0024	-0.3415***	0.0025	-0.4053***	0.0033
3rd grade[a, b]	0.6550***	0.0036	-0.1861***	0.0034	No test administered	No test administered
4th grade[a, b]	0.1509***	0.0050	-0.1402***	0.0050	No test administered	No test administered
5th grade[a, b]	0.0221***	0.0050	0.2407***	0.0052	0.4462***	0.0166
6th grade[a, b]	0.0168***	0.0049	-0.3039***	0.0049	No test administered	No test administered
7th grade[a, b]	0.3009***	0.0050	-0.3963***	0.0049	No test administered	No test administered
8th grade[a, b]	0.1355***	0.0050	-0.3139***	0.0049	0.1625***	0.0166
9th grade[a, b]	No test administered	No test administered	No test administered	No test administered	0.3074***	0.0232
10th grade[a, b, c]	No test administered	No test administered	Omitted	Omitted	0.2003***	0.0167
11th grade[a, b, c]	Omitted	Omitted	No test administered	No test administered	0.1528***	0.0176
American Indian[a]	-0.2212***	0.0061	-0.1707***	0.0065	-0.3579***	0.0087
Asian American[a]	-0.0012	0.0047	-0.0333***	0.0047	-0.1030***	0.0062
Hispanic[a]	-0.2846***	0.0041	-0.2040***	0.0042	-0.3968***	0.0057
Black[a]	-0.3945***	0.0034	-0.3216***	0.0036	-0.6216***	0.0048
Female[a]	-0.0127***	0.0020	0.1461***	0.0021	-0.0816***	0.0028
Receives special education[a]	-0.7104***	0.0026	-0.7175***	0.0028	-0.8806***	0.0037
Homeless[a]	-0.2416***	0.0095	-0.2317***	0.0098	-0.2567***	0.0148
Primary language other than English[a]	-0.1037***	0.0035	-0.2226***	0.0036	-0.3224***	0.0050
Pilot district[a]	-0.0094**	0.0033	0.0241***	0.0035	-0.0014	0.0047
Alternative Learning Center and Programs[a]	-0.6623***	0.0104	-0.4578***	0.0118	-0.6837***	0.0140
Student assessed in Fiscal Year 2013[a]	-0.2484***	0.0044	0.1613***	0.0045	0.0290***	0.0052
Student assessed in Fiscal Year 2014[a]	-0.2033***	0.0041	0.0519***	0.0039	0.0906***	0.0052
Student assessed in Fiscal Year 2015[a]	-0.2448***	0.0041	-0.0045	0.0037	0.0700***	0.0052
Student assessed in Fiscal Year 2016[a]	-0.2648***	0.0041	0.0152***	0.0037	0.1601***	0.0054
Student assessed in Fiscal Year 2017[a]	-0.2856***	0.0041	0.0365***	0.0037	0.1051***	0.0054
Student assessed in Fiscal Year 2018[a, c]	-0.3323***	0.0041	Omitted	Omitted	0.0655***	0.0053
Graduation rate	0.0451***	0.0088	0.0500***	0.0090	0.0523***	0.0124
Dropout rate	-0.0090***	0.0005	-0.4626***	0.0277	-0.7251***	0.0349
Attendance rate	1.9993***	0.0591	0.4811***	0.0658	0.8270***	0.0879
Student-to-teacher ratio	0.0016*	0.0007	0.0079***	0.0007	0.0043***	0.0010

Continued on next page.

Exhibit B.2: Estimated Coefficients and Standard Errors from Primary Regression Model, Fiscal Years 2011 through 2018 (continued)

	Math		Reading		Science	
	Coefficient	Standard Error	Coefficient	Standard Error	Coefficient	Standard Error
Student-to-classroom-aide ratio	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Percentage of students qualifying for free or reduced-price lunch (per student in the school)	-0.0053***	0.0001	-0.0050***	0.0001	-0.0068***	0.0001
Net tax capacity per student in school districts[d, e]	2.4974***	0.2704	1.7122***	0.2939	2.8252***	0.3920
Compensatory revenue per student in the school[e, f]	0.0065***	0.0009	0.0012	0.0008	-0.0023*	0.0010
General education revenue per student in the district[e, f]	-0.0309***	0.0019	-0.0012	0.0021	-0.0189***	0.0028
English learner revenue per English learner student in the district[e, f]	0.0000	0.0001	-0.0001	0.0001	0.0002	0.0002
Number of English learners per student in the district	0.1270***	0.0158	0.0474**	0.0163	-0.1033***	0.0244
Average daily membership[g]	-0.0002	0.0001	-0.0005***	0.0001	0.0027***	0.0002
Charter school[a]	-0.3042***	0.0203	0.0446*	0.0211	-0.0862**	0.0282
Cooperative[a]	-0.3710***	0.0244	0.0533*	0.0262	-0.0936**	0.0348
Student attended school in a rural county[a, h]	0.0435***	0.0047	0.0462***	0.0048	0.0303***	0.0062
Student attended school in Hennepin or Ramsey county[a]	0.0546***	0.0050	0.0908***	0.0052	0.0755***	0.0067
Student attended school in a suburban county[a, i]	0.0642***	0.0045	0.0738***	0.0046	0.0943***	0.0061
Student attended school in a smaller metropolitan statistical area[a, j]	-0.0082	0.0046	0.0280***	0.0047	-0.0231***	0.0062
Previous achievement level was "does not meet standards"[a]	-0.9528***	0.0040	-0.8645***	0.0046	-0.9758***	0.0246
Previous achievement level was "partially meets standards"[a]	0.2265***	0.0039	0.2117***	0.0047	0.2619***	0.0266
Previous achievement level was "meets standards"[a]	1.3404***	0.0046	1.1916***	0.0051	1.4175***	0.0480
Previous achievement level was "exceeds standards"[a]	2.1206***	0.0098	1.7841***	0.0090	1.5616***	0.0788
Constant[k]	-0.3991***	0.0618	0.6596***	0.0676	0.5052***	0.0912

NOTES: One asterisk (*) indicates less than a 0.05 probability of no relationship with the variable called "Meets standards." Two asterisks (**) indicate that the probability of no relationship is less than 0.01. Three asterisks (***) indicate that the probability is less than 0.001. No asterisk indicates that there is greater than a 0.05 probability of no relationship and is therefore, not statistically significant. "Standard error" is an indicator of the precision of an estimate. In general, a smaller standard error indicates a more precise estimate; a larger standard error indicates a less precise estimate. We determined when a student meets standards by analyzing student scores from the Minnesota Comprehensive Assessments, the state's standardized tests. Achievement levels of exceeding, meeting, and partially meeting standards were grouped together as meeting standards for this analysis.

Continued on next page.

Exhibit B.2: Estimated Coefficients and Standard Errors from Primary Regression Model, Fiscal Years 2011 through 2018 (continued)

[a] Binary variable where the value equals one (1) when the observation fits the category and otherwise equals zero (0).

[b] Represents a student's grade level when taking the Minnesota Comprehensive Assessments (MCAs). We included grade variables because an expert in education research said the fact that a relatively small number of students take the MCAs in some years is a source of variation in test performance across grades.

[c] The variable was omitted in at least one of the regressions. If the variable had been included, then the sum of all the variables in the group (i.e., all the grade variables) would duplicate the "Constant" variable. The solution to avoid such duplication is to omit one of the variables, as is represented here.

[d] In millions of dollars.

[e] Amounts from different years have been adjusted for inflation in Fiscal Year 2018 dollars.

[f] In thousands of dollars.

[g] In thousands of students.

[h] OLA defined as Minnesota counties that were not part of a metropolitan statistical area, according to the U.S. Office of Management and Budget as of 2013.

[i] OLA defined as Anoka, Carver, Dakota, Scott, and Washington counties. In estimating coefficients on Minnesota regions, the analysis did not include a separate variable for students living in the exurban counties of Chisago, Isanti, Le Sueur, Mille Lacs, Sherburne, Sibley, and Wright. These exurban counties were part of the Minneapolis-St. Paul-Bloomington metropolitan statistical area, according to the U.S. Office of Management and Budget in 2013. We could predict the exurban counties using the other county groups listed in this exhibit.

[j] OLA defined as counties located in metropolitan statistical areas (MSAs), according to the U.S. Office of Management and Budget as of 2013, outside the Minneapolis-St. Paul-Bloomington MSA. They were Carlton and St. Louis counties (Duluth MSA); Clay County (Fargo, ND, MSA); Polk County (Grand Forks, ND, MSA); Houston County (La Crosse, WI, MSA); Blue Earth and Nicollet counties (Mankato-North Mankato MSA); Dodge, Fillmore, Olmsted, and Wabasha counties (Rochester MSA); and Benton and Stearns counties (St. Cloud MSA).

[k] The constant is a number that can be used to calculate the probability of meeting academic standards when all other variables equal zero. Since there is no situation where all values of the other variables listed above equal zero, it is of limited use. The values for the constant are here to allow replication of results.

SOURCES: Office of the Legislative Auditor, analysis of data from the Minnesota Department of Education; U.S. Bureau of Labor Statistics, *Consumer Price Index*, https://data.bls.gov/pdq/SurveyOutputServlet, accessed on December 23, 2019; and Executive Office of the President, U.S. Office of Management and Budget, *Revised Definitions of Metropolitan Statistical Areas, Micropolitan Statistical Areas, and Combined Statistical Areas, and Guidance on Uses of the Definitions of These Areas*, OMB Bulletin No. 13-01, February 28, 2013.

Although our primary focus was to identify the correlation between students qualifying for free or reduced-price lunch and meeting academic standards, readers may find interesting a few of the other variables' estimated coefficients. At the same time, for a number of variables, we either did not have data or we did not employ methods that would better ensure the estimates' reliability. We discuss this briefly in the final section of the appendix.

- **Homeless students.** The coefficients found in Exhibit B.2 for this variable indicate that homeless students were less likely than other students to have met MCA academic standards. A homeless student is more likely to be chronically absent, which can lead to lower academic success.[7]

[7] Minnesota Department of Education, "Homework Starts with Home," https://education.mn.gov/MDE/fam/home/, accessed December 20, 2019.

- **Students receiving special education.** The variable on students receiving special education showed that these students were also less likely than other students to meet academic standards in all three subject areas. Our results are not unique to Minnesota. Gaps in achievement exist for students enrolled in special education in other parts of the nation.[8]

- **Nonwhite students.** Students of color are less likely to meet academic standards in comparison with white students, in most instances.[9] Other research on the gap in achievement between black and white students corroborates our results. Some of the research states that racial segregation "concentrates minority students in high-poverty schools," which also correlates with the achievement gap.[10]

- **Previous MCA results.** Students who performed better on their previous MCA were more likely to meet standards on a current test. Other studies have found that previous test scores tend to increase the accuracy in estimating current test scores.[11]

- **Pilot program school districts and programs.** School districts in Minnesota's pilot compensatory revenue program do not have the same restrictions as traditional school districts for allocating compensatory revenue. Regression results regarding the variable for pilot programs were mixed. The variable is associated with a lower probability of meeting standards in math and a higher probability of meeting standards in reading; it was not statistically significant in science. However, one cannot use our analysis to estimate the direct impact of pilot districts on student MCA achievement levels. The estimated coefficients for the pilot districts do not include the effect of compensatory revenue on MCA achievement levels. This is because compensatory revenue is a separate variable in our analysis.

- **Alternative learning centers and programs.** These centers and programs are designed for students having difficulty in the traditional education system.[12] Based on our results, students in alternative learning centers were associated with lower MCA achievement levels. Like the variable on pilot school districts, however, the variable on alternative learning centers and programs does not allow us to draw conclusions about the direct impact of these centers or

[8] Deb A. Albus, Kristin K. Liu, Martha L. Thurlow, and Sheryl S. Lazarus, *2016-17 Publicly Reported Assessment Results for Students with Disabilities and ELs with Disabilities,* NCEO Report 411 (Minneapolis: University of Minnesota, National Center on Educational Outcomes, 2019), 25.

[9] In an exception, our results showed no statistically significant difference in the achievement level on the math MCA between white students and Asian-American students, net of other factors.

[10] Sean F. Reardon, Ericka S. Weathers, Erin M. Fahle, Heewon Jang, and Demetra Kalogrides, *Is Separate Still Unequal? New Evidence on School Segregation and Racial Academic Achievement Gaps,* Center for Education Policy Analysis Working Paper No. 19-06 (Palo Alto: Stanford University, 2019), 1.

[11] Patrick J. McEwan, "Quantitative Research Methods in Education Finance and Policy," in *Handbook of Research in Education Finance and Policy, Second Edition,* eds. Helen F. Ladd and Margaret E. Goertz (New York: Routledge, 2015), 87-92.

[12] *Minnesota Statutes* 2019, 124D.68, subd. 1.

programs on students' MCA results. This is because the variable does not include the effect of compensatory revenue—an effect that could contribute to the impact of the centers and programs on MCA achievement levels.

- **Percentage of total enrollment that qualifies for subsidized lunch.** This variable accounts for the concentration of poverty within a school. Studies elsewhere have found that students within schools with high poverty rates tend to perform more poorly than students attending schools with low concentrations of poverty.[13] Our estimated coefficients in Exhibit B.2—for example, -0.0053 on the math assessment for the variable "Percentage of students qualifying for free or reduced-price lunch (per student in the school)"—indicated a similar tendency.

- **Compensatory revenue per student.** We included a variable representing the amount of compensatory revenue per student. Although we included the variable, we cannot conclusively state whether or to what extent compensatory revenue per student correlates with higher achievement levels. With currently available data, it would be difficult to determine whether low-income students who received services paid for with compensatory revenue performed better than low-income students who did not receive those services. In other words, we had no control group. Given this limitation, we estimated that a one-unit increase in compensatory revenue per student was associated with higher achievement levels in math and lower achievement levels in science; it had no statistically significant correlation with reading achievement.

Results from Alternate Approaches

Because other factors unaccounted for in our primary model might change the correlation between students qualifying for free or reduced-price lunch and achievement levels, we tested alternate approaches. The purpose of the tests was to determine whether alternate approaches would achieve the same result as the primary model—that students qualifying for subsidized lunch are an appropriate but limited proxy for students who do not meet MCAs' academic standards.

In one alternative, we redefined who to include in the definition of students who "meet academic standards." In a second alternative, we used different specifications for certain independent variables. In a third alternative, we used different regression techniques to address possible statistical problems. We define the three alternatives below.

Defining Variables Differently

In our first alternate approach, we altered the definition of student achievement. As stated earlier, MDE groups MCA scores into one of four academic achievement levels: "exceeding" standards, "meeting" standards, "partially meeting" standards, and "not meeting" standards. We tested an alternate definition of achievement levels to determine whether it would eliminate the correlation between students who received subsidized lunch and students who met MCA standards. For our alternate approach, we

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[13] Reardon et al., *Is Separate Still Unequal?*, 1.

defined "not meeting standards" to include students who *partially* met standards, along with students who had not met standards. By contrast, in our primary model, students not meeting the standards had been in a group by themselves, and students who partially met the standards had been grouped with students who exceeded or met the standards.

Our analysis with the alternate definition yielded results similar to those from our primary model. Both sets of results indicate that students who qualified for subsidized lunch were less likely than others to have met academic achievement standards. The probability that there is actually no correlation at all is also quite low, similar to results from our primary model.

Altering Variables

In a second alternative to the primary model, we changed certain variables. Namely, in our alternate approach, we analyzed the dollar amount of compensatory revenue per school district or charter school instead of compensatory revenue on a per-student basis. We also converted the binary variables for grade and school year into a variable that listed the grade for each student and a separate variable that listed the year that the student attended the public school.

Alternate sets of variables in a regression analysis can yield different results, but that was not the case here. Even with the altered variables, our results showed that students qualifying for subsidized lunch were less likely to meet academic standards.

Using Different Regression Techniques

In our final alternate approach, we tested additional regression techniques to address statistical issues that an ordinary probit model does not. As described earlier in the Methodology section, we used three alternative regression models: a probit with random effects, an ordered probit, and a bivariate probit. Regardless of the alternate model we used, our results were the same. Students qualifying for subsidized lunch were less likely than other students to meet academic standards. From the alternate regressions, the estimated coefficients among the math, reading, and science MCA results ranged from -0.3 to -0.7. The negative coefficients meant that a student on subsidized lunch was less likely to meet the MCA academic standards.[14] Moreover, the probability that no such relationship existed was very low, at less than a 1 in 10,000 chance.

Limitations

Although we used different approaches to gather more complete information, our analyses still had limitations. In the following section, we describe three types of limitations: variables, regression modeling, and the use of MCA results as measures of academic standards.

[14] For all of the regressions, this was true if every other variable was held constant and the only change was the variable "Qualifies for free or reduced-price lunch." The ordered probit was slightly different in that a negative coefficient meant that the student was less likely to be at a higher achievement level.

Variables. Even though we used more than 45 variables in our primary model, we did not account for a number of others that could potentially explain students' MCA results. No research study can both identify and obtain data for all relevant variables. We identified additional factors that could potentially affect the probability of not meeting academic standards. Below are examples for which we did not have data.

- The effect teacher quality has on student performance.
- The effect of school materials, such as computers, books, and software.
- The effect of parental involvement on a student's performance.
- Whether the school used best practices to reach students who were behind in school.[15]

Regression Models. Our alternative approaches separately accounted for three issues: (1) uniqueness of students or individual differences that are unaccounted for with any variable in the dataset; (2) the use of four ordered categories for academic achievement—exceeding standards, meeting standards, partially meeting standards, or not meeting standards; and (3) the variable indicating the individual student qualified for free or reduced-price lunch could potentially be an outcome variable (in addition to the known outcome variable called "Meets academic standards"). However, more statistical modeling issues remained. Below are a few examples.

- The type of regression we chose can identify a correlation between students qualifying for free or reduced-price lunch and lower achievement scores, but it cannot determine causation. Other techniques may isolate a causal effect to qualifying for subsidized lunch, but they were not amenable to our research question and data.[16]
- Additional alternative statistical models could have addressed other issues with the data. One issue is that a school-level variable, such as the percentage of students qualifying for subsidized lunch, could depend upon other factors. We used statistical models that corrected for some, but not all, modeling issues. Whether to use a model that adjusts for such a statistical issue partly depended upon judgment about whether the conclusions would substantially change under that model.
- Another potential modeling issue is that obtaining reliable results may depend upon simultaneously addressing in one regression model all three of the statistical issues listed earlier. Instead, we addressed a single statistical issue at a time by using three separate alternative models—a random effects probit, an ordered probit, and a bivariate probit. The fact that all three statistical issues

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[15] In OLA's 2019 survey of school districts and charter schools, we collected school-district (and charter-school) level data on certain best practices. However, due to a lack of time, we could not prepare and merge these survey data with data in the regression analysis.

[16] For example, one possible technique would have matched students qualifying for subsidized lunch with students that did not qualify. We did not use a matching technique because of expected difficulties in finding close matches within the two groups, since the two populations would differ on many demographic variables.

were simultaneously present in our data, however, is a limitation of our analysis. Newer techniques can simultaneously handle multiple statistical issues like the ones listed above. However, it remains uncertain whether combining statistical issues into one regression would alter our conclusion, given the high levels of statistical significance to the correlation. For example, an ordered probit with a random effects model can estimate coefficients, but it is uncertain whether doing so would change our conclusion that qualifying for free or reduced-price lunch increases the probability of not meeting academic standards.

MCA Achievement Levels. Limitations pertaining to using MCA achievement levels as a measurement of academic success include the following.

- Our analysis cannot be used to explain the reasons why students fail to meet MCA's performance standards. As the Office of the Legislative Auditor's 2017 *Standardized Student Testing* report describes, standardized tests "only measure student performance, they cannot explain it."[17] Data on MCA achievement levels do not provide a reason for each student's performance. They do not reveal a student's circumstances, such as whether the student was homeless during the school year. Instead, the achievement levels provide information on student performance as measured by academic, grade-level standards.

- MCA scores measure a student's achievement at a single point in time. Consequently, we did not measure a student's academic progress over the years. For students who had taken MCAs in prior years though, we did include prior achievement levels in our regression, as discussed earlier in the Primary Model Results section. Our regression models found a correlation between students' prior and current achievement levels. However, it is inappropriate to infer that students' current success meant they had made progress over time on meeting MCA standards.

[17] Office of the Legislative Auditor, Program Evaluation Division, *Standardized Student Testing* (St. Paul, 2017), 4.